The Physics of Flight
100 Years Since the Wright Brothers

by

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Sir George Cayley, 1773-1857

• Showed lift is proportional to velocity squared and \( \sin \alpha \), 1804
• Wrote a three-part paper on “Aerial Navigation,” 1809-1810
• Designed the first successful glider
A replica of Caley’s glider, flown by Derek Piggott, 1973
Otto Lilienthal (1848-1896)

- Measured the lift and drag of a wing
- Made over 2000 flights in a glider, some as far as 350m
- Wrote a book “The flight of birds as the basis for the art of flying,” 1886.
- First person killed in an aircraft accident, 1896.
Orville and Wilbur Wright (1871-1948; 1867-1912)

• Knew of Cayley and Lilienthal’s work
• Made one of the first wind tunnels, 1901
• Invented wing warping as a method of roll control, 1902
Wright Brothers’ 1902 Glider
first full 3-axis controlled flight
Orville Wright, Dec. 17, 1903
first controlled powered flight
Status of Aerodynamic Theory in the Early 1900s

- All of the basic equations of fluid mechanics were known.

- However, no acceptable theory existed for the lift force on a wing.

- Worst than that, the existing theories predicted that the lift was exactly zero.
Bernoulli’s Theorem and the Lift on a Wing

\[
\frac{1}{2} \rho_m U^2 + p = \text{constant}
\]

\( \rho_m \) = mass density, \( U \) = velocity

\( p = \frac{F}{A} \) is the pressure

\( F = \text{force}, \ A = \text{Area} \)

The usual argument:

Since \( l_1 > l_2 \) it follows that \( U_1 > U_2 \) so \( p_1 < p_2 \). Therefore, \( F_2 > F_1 \). (WRONG). One cannot assume that the stagnation point (where the streamlines separate) is exactly at the leading edge. One must solve for the entire flow pattern over the wing, which varies considerably with the angle of attack.
Some Key Concepts

Vorticity
\[ \vec{\omega} = \vec{\nabla} \times \vec{U} \]

Circulation
\[ \Gamma = \int_c \vec{U} \cdot d\vec{\ell} \]

Theorem: \[ \vec{\nabla} \cdot \vec{\omega} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{U}) = 0 \]
Vortex lines are continuous

Theorem: \[ \Gamma = \int_c \vec{U} \cdot d\vec{\ell} = \int_s \vec{\nabla} \times \vec{U} \cdot d\vec{A} \]

\[ \Gamma = \int_s \vec{\omega} \cdot d\vec{A} \]
Basic Equations and Assumptions

- Continuity equation
  \[ \frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{U}) = 0 \]

- Navier Stokes equation
  \[ \rho_m \left[ \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla) \vec{U} \right] = -\nabla p + \rho_m \nu \left[ \nabla^2 \vec{U} + \frac{1}{3} \nabla (\nabla \cdot \vec{U}) \right] \]

- The assumption of incompressibility
  If \( \rho_m \) = constant, then
  \[ \nabla \cdot \vec{U} = 0 \]

- The vorticity equation
  \[ \frac{\partial \vec{\omega}}{\partial t} = \nabla \times (\vec{U} \times \vec{\omega}) + \nu \nabla^2 \vec{\omega} \]
  Convection  Diffusion

- Reynolds Number
  \[ R_N = \frac{|\text{convection}| \cdot UL}{|\text{diffusion}| \cdot \nu} \]
## Reynolds Number

**Typical values (at sea level)**

<table>
<thead>
<tr>
<th></th>
<th>U</th>
<th>L</th>
<th>$R_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commercial Jet</td>
<td>600 mph</td>
<td>20 ft</td>
<td>$1.1 \times 10^8$</td>
</tr>
<tr>
<td>Light plane</td>
<td>100 mph</td>
<td>5 ft</td>
<td>$4.7 \times 10^6$</td>
</tr>
<tr>
<td>Glider</td>
<td>60 mph</td>
<td>3 ft</td>
<td>$1.6 \times 10^6$</td>
</tr>
<tr>
<td>Model airplane</td>
<td>40 mph</td>
<td>8 in</td>
<td>$2.5 \times 10^5$</td>
</tr>
<tr>
<td>Seagull</td>
<td>20 mph</td>
<td>4 in</td>
<td>$6.2 \times 10^4$</td>
</tr>
<tr>
<td>Butterfly</td>
<td>5 mph</td>
<td>1 in</td>
<td>$3.9 \times 10^3$</td>
</tr>
<tr>
<td>Housefly</td>
<td>5 mph</td>
<td>0.3 in</td>
<td>$1.2 \times 10^2$</td>
</tr>
</tbody>
</table>
Basic Equations and Assumptions (con’t)

• The inviscid assumption, \( R_N \ll 1 \)
  If \( \nu = 0 \), then
  \[
  \rho_m \left[ \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \vec{V}) \vec{U} \right] = -\vec{V} p \quad \text{(Euler’s equation)}
  \]

• Kelvin’s theorem
  If \( \rho_m = \text{constant} \) and \( \nu = 0 \), then
  \[
  \Gamma = \int_s \vec{\omega} \cdot d\vec{A} = \text{constant}
  \]

• Velocity potentials
  If \( \Gamma = 0 \) in the upstream flow, then \( \vec{\omega} = \vec{V} \times \vec{U} = 0 \) at all points in the flow. It follows then that
  \[
  \vec{U} = \vec{V} \Phi
  \]

• Laplace’s equation
  From the continuity equation, \( \vec{V} \cdot \vec{U} = 0 \), one then has
  \[
  \nabla^2 \Phi = 0, \quad \text{or} \quad \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0
  \]
Complex Potentials

If \( z = x + iy \), any analytic complex function \( F(z) = \Phi(x, y) + i\Psi(x, y) \) provides a solution to Laplace’s equation

\[
\nabla^2 \Phi = 0 \quad \text{and} \quad \nabla^2 \Psi = 0
\]

Examples:

- **Uniform flow**
  - \( U_0z \)

- **At an angle \( \alpha \)**
  - \( U_0ze^{-i\alpha} \)

- **Dipole**
  - \( \frac{\mu_0}{z} \)

- **Vortex**
  - \( i\frac{\Gamma_0}{2\pi} \ln z \)
Flow Around a Cylinder

- Horizontal flow
  \[ U_0 \left( z + \frac{a^2}{z} \right) \]

- At an angle \( \alpha \)
  \[ U_0 \left( z + \frac{a^2}{z} \right) e^{-i\alpha} \]

- With circulation
  \[ U_0 \left( z + \frac{a^2}{z} \right) + \frac{i\Gamma_0}{2\pi} \ell n \left( \frac{z}{a} \right) \]
The Blasius Force Equation

\[ F_x - iF_y = i \rho_m \frac{\partial m}{\partial y} \int_c W^2 \, dz , \]

where \( W(z) = dF/dz = U_x - iU_y \) is the complex velocity.

\[ W(z) = U_0 + i \frac{\Gamma_0}{2\pi} \sum_n \frac{1}{z^n} \]

\[ \int_c W^2 \, dz = 2\pi i \sum \text{Res}(W^2) = -i2U_0 \Gamma_0 \]

\[ F_x = 0 \]

\[ F_y = \rho_m U_0 \Gamma_0 \]
The Joukowski Transformation, 1910

The Joukowski transformation

\[ z' = z + \frac{c^2}{z} \]

where \( z' = x' + iy' \),

transforms a cylinder into a flat plate.
Flow Around a Flat Plate Airfoil

Since \( \Gamma_0 = 0 \), by the Blasius theorem \( F_y = 0 \).

The dilemma: Since the upstream vorticity is zero the circulation must be zero, so there can be no lift.

But a flat plate airfoil produces lift, so there must be circulation.
The Kutta-Joukowski Condition, 1910

The flow must be smooth and continuous at the trailing edge. Requires a circulation, \( \Gamma_0 = 4\pi U_0 a \sin \alpha \).

\[
F_y = \rho_m U_0 \Gamma_0
\]

\[
L = \left( \frac{1}{2} \rho_m U^2 \right) A 2\pi \sin \alpha
\]

\[
C_L = \frac{L}{\left( \frac{1}{2} \rho_m U_0^2 \right) A} = 2\pi \sin \alpha
\]
The Joukowski Family of Airfoils

\[ C_L = 2\pi \left(1 + 0.77 \frac{t}{\ell}\right) \sin(\alpha + \beta_0), \quad \beta_0 = 2h / \ell \]
Comparison with Experimental Data

\[ C_L = 2\pi \sin(\alpha + \beta_0) \]

\[ C_L \]

\[ \alpha \text{ (deg)} \]

\[ \beta_0 \]

Airfoil

Stall
Wind Tunnel Observations

\[ \alpha = 5^\circ \]  
Kutta condition satisfied

\[ \alpha = 10^\circ \]  
Slight flow separation

\[ \alpha = 15^\circ \]  
Complete flow separation (stall)
The Maximum Lift Coefficient

$C_L, \max$

$\alpha + \beta_0$, Angle of Attack

Fowler Flap
Cambered Airfoil
Symmetrical Airfoil
Curved Plate
Flat Plate

$2\pi$
Origin of the Circulation

\[ \frac{\partial \tilde{\omega}}{\partial t} = \tilde{\nabla} \times (\tilde{U} \times \tilde{\omega}) + v \nabla^2 \tilde{\omega} \]

\[ \frac{1}{L} \quad \frac{1}{L^2} \]
Where is the Vorticity?
(In the boundary layer)

\[ \Gamma_0 = \int_S \vec{\omega} \cdot d\vec{A} \]

No Slip Condition

Boundary Layer

\[ f = v \rho_m \frac{\partial U_x}{\partial y} A \]
Finite Wing Span Effects
(continuity of \( \vec{\omega} \))

Note: \( \Gamma_1 = \Gamma_2 = \Gamma_0 \), vortex trails cannot be avoided.
Vortex Trails
The Induced Downflow at the Wing

Note: The downflow velocity at the wing is exactly $\frac{1}{2}$ of the downstream value (for a straight wing)
Induced Drag, $D_i$

Upstream

At the wing

$D_i = \text{component of } L \text{ in the } U_0 \text{ direction}$

$\delta \cap \frac{U_y}{U_0} = \text{angle of the downflow at the wing}$
The Elliptical Wing Theorem
Prandtl, 1918-1919

\[ L = \rho_m U_0 \int_{-s}^{s} \Gamma(z) \, dz \]

\[ D_i = -\frac{\rho_m}{4} \int_{-s}^{s} \int_{-s}^{s} \frac{d\Gamma \, \Gamma(z')}{dz \, dz'} \, dz \, dz' \]

Problem: Minimize \( D_i \), while holding \( L \) constant

Answer: \( \Gamma(z) = \Gamma_c \left[ 1 - \left(\frac{z}{s}\right)^2 \right]^{1/2} \), i.e., an ellipse
The Total Drag Force

Induced Drag

\[ D_i = \frac{1}{4\pi} \left( \frac{A}{s^2} \right) \frac{L^2}{\left( \frac{1}{2} \rho_m U^2 A \right)} \]

Viscous Drag

\[ D_f = \frac{1}{2} \rho_m U^2 A C_{D_f} \]

Note, \( A/s^2 = 1/\text{Aspect ratio} \)
Some Elliptical Wings
Viscous Drag Reduction
Laminar Flow Airfoils, NACA 1930s
First Use of a Laminar Flow Airfoil, P-51, 1940
Compressibility Effects

Shock Wave

\[ v_s \sim p^\delta, \quad \delta = 1/5 \]

Mach Number
(Ernst Mach, 1889)

\[ M = \frac{U_0}{V_s} \]

Mach Cone

\[ \sin \beta = M \]

Detached Shock

Strong Shock

\[ M_1 > 1 \]
\[ M_2 < 1 \]

Attached Shock

Weak Shock

\[ M_1 > 1 \]
\[ M_2 > 1 \]

A-D03-171-1
Thin Wing Theory  
(Theodor von Karman, 1940s)

Laplace’s equation modified for compressibility effects

\[
(1-M^2) \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0
\]

\( M < 1 \)  
Elliptical differential equation  
\( M > 1 \)  
Hyperbolic differential equation

Solution: transform to \( M = 0 \)

\[
x' = \frac{x}{\sqrt{1-M^2}}
\]

Solution: wave equation

\[
\Phi = f \left( y - \frac{1}{\sqrt{M^2 - 1}} x \right)
\]
The Flat Plate Airfoil

Subsonic (M < 1)

\[ C_L = \frac{2\pi}{\sqrt{1-M^2}} \sin \alpha \]

Supersonic (M > 1)

\[ C_L = \frac{4\alpha}{\sqrt{M^2 - 1}} \]

Note the change in the center of pressure, from \( \ell/4 \) to \( \ell/2 \).
The Critical Mach Number

- Maximum Local Velocity Equal to Sound Speed
  - $M = 0.72$

- Supersonic Flow
  - $M = 0.77$
  - Shock Wave
  - Subsonic

- Drag Coefficient, $C_D$

- Critical Mach Number

- Drag Due to Shock Wave

- Mach Number, $M$
  - 0
  - 1.0
  - 2.0
  - 3.0
A Shock at the Critical Mach Number
Sweepback
(Busemann, 1935)

Sweepback increases the critical Mach number

First use of sweepback
Me-262, 1941
Supercritical Airfoil
(Whitcomb, 1971)

Conventional Airfoil

Supercritical Airfoil

Boeing 777 ($M_c = 0.85$)
A Wind Tunnel in Your Computer

Design your own airfoils
Smooth coordinates
Your choice of Reynolds numbers
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Predict the performance of airfoils
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Your choice of angles of attack
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Boundary layer analysis with
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