Continuum Radiation Associated With Low-Energy Electrons in the Outer Radiation Zone

D. A. Gurnett and L. A. Frank

Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242

A weak nonthermal continuum radiation is generated by the earth's magnetosphere in the frequency range from about 500 Hz to greater than 100 kHz. During magnetically disturbed periods the intensity of this continuum radiation increases significantly, by as much as 20 dB during large disturbances. In this paper we present a series of observations obtained by the Hawkeye 1 and Imp 8 spacecraft during a period of greatly enhanced continuum radiation intensity which occurred from October 14-21, 1974. The enhanced continuum radiation intensities observed during this event are found to be closely correlated with the injection of very intense fluxes of energetic, ~1-30 keV, electrons into the outer radiation zone. Direction-finding measurements of the continuum radiation observed during this event show that the radiation is primarily coming from the dawn side of the magnetosphere, in agreement with the observed dawn-dusk asymmetry in the 1- to 30-keV electron distribution. These results suggest that the continuum radiation may be generated by a coherent plasma instability involving relatively low-energy, ~1 to 30 keV, electrons rather than by gyrosynchrotron radiation from very energetic, 200 keV to 1 MeV, electrons as has been previously suggested.

INTRODUCTION

Two principal types of radio emissions can be distinguished coming from the earth's magnetosphere at frequencies above the local plasma frequency: auroral kilometric radiation and continuum radiation. Auroral kilometric radiation consists of very intense and highly variable emissions with peak intensities in the frequency range from about 100 to 300 kHz [Gurnett, 1974]. This radiation is closely associated with the occurrence of aurora and is generated at altitudes of 1-2 Re in the nighttime auroral zones [Kurth et al., 1975]. Continuum radiation is a much weaker and slowly varying emission which extends over a broad range of frequencies, from as low as 500 Hz to greater than 100 kHz. Continuum radiation is thought to be generated at much higher altitudes in the outer radiation zone [Gurnett, 1975]. Figure 1 shows typical spectrums of these two types of radio emissions, as observed by a spacecraft about 30 Re from the earth.

Recently, Kaiser and Stone [1975] have identified a third type of radio emission which appears to be generated in the day side auroral region. This radiation is very weak, with intensities only slightly above the galactic background, and occurs in a narrow frequency band with peak intensity slightly below 200 kHz. It seems likely that this weak narrow band emission is simply the day side analogue of the much more intense nighttime auroral kilometric radiation. However, at this time the exact relationship of this weak narrow band emission to auroral kilometric radiation has not been clearly established.

Continuum radiation from the earth's magnetosphere was first identified by Brown [1973] at frequencies from about 30 to 100 kHz using radio measurements from the Imp 6 satellite. Further studies by Gurnett [1975] have shown that the continuum radiation comprises two components: one that is permanently trapped within the low-density regions of the magnetospheric cavity at frequencies below the solar wind plasma frequency and one that can propagate freely away from the earth at frequencies above the solar wind plasma frequency. Direction-finding measurements show that the continuum radiation is generated in a broad region on the dawn side and early afternoon side of the magnetosphere, as is shown in Figure 1. Whereas the intensity of auroral kilometric radiation sometimes varies by as much as 60-80 dB in only a few minutes, the intensity of the continuum radiation varies over a much smaller range, usually not more than 20 dB, and on a longer time scale, usually several hours or more. Frankel [1973] has shown that enhancements in the continuum radiation intensity, by as much as 20 dB above the quiescent intensity, occur during periods of geomagnetic disturbance. These periods of enhanced continuum radiation intensity sometimes last for several days.

Frankel [1973] proposed that the continuum radiation is generated by gyrosynchrotron radiation from energetic electrons, 200 keV to 1 MeV, injected into the outer radiation zone during the magnetic storm. Spectrums of the gyrosynchrotron radiation intensity computed by Frankel for a typical distribution of high-energy electrons injected into the outer radiation zone during a magnetic storm are in reasonably good qualitative agreement with the observed spectrum, but the absolute intensity is about a factor of 5-15 too small.

The purpose of this paper is to present a series of observations from the Hawkeye 1 satellite during a period of enhanced continuum radiation intensity which occurred from October 14-21, 1974. This event commenced during a geomagnetically disturbed period following a magnetic storm on October 14, 1974. The enhanced continuum radiation intensities occur in association with intense fluxes of low-energy electrons, ~1-30 keV, injected deep into the outer radiation zone during this storm. The energy range of the electrons responsible for the continuum radiation and possible mechanisms for producing the radiation are considered.

The observations used in this study were gained primarily from the Hawkeye 1 spacecraft, which was launched on June 3, 1974. Hawkeye 1 is in a highly eccentric polar orbit with initial perigee and apogee geocentric radial distances of 6847 km and 130,856 km, respectively, an orbital inclination of 89.7°, and a period of 49.94 hours. The initial argument of perigee is 274.6°, so that the apogee is located almost directly over the north pole. The plasma wave experiment on Hawkeye 1 uses an electric dipole antenna with a tip-to-tip length of
Plate 1. A series of electron energy-time spectrograms gained with the Lepeda instrument on Hawkeye 1 during the October 14-21 event. These spectrograms are for inbound passes through the outer radiation zone at local morning. Note the substantial increase in the electron intensities for $1 \leq E \leq 30$ keV over $4 < L < 8$ commencing with orbit 63, after the onset of the enhanced continuum radiation.
Plate 2. The spectrograms corresponding to those in Plate 1 for the outbound passes at local evening. Note the substantial decreases of both the electron energy and the intensities and lesser depth of penetration into the outer zone compared to those for the corresponding passes at local morning in Plate 1.
Fig. 1. Representative spectrums of the galactic background, auroral kilometric radiation, and the trapped and escaping continuum radiation as would be observed by a satellite about 30 $R_E$ from the earth. The continuum radiation is generated in a broad region beyond the plasmapause in the local morning and local noon sectors of the magnetosphere.

42.45 m for electric field measurements and a search coil magnetometer for magnetic field measurements. Electric field spectrum measurements are made in 16 frequency channels extending from 1.78 Hz to 178 kHz, and the magnetic field spectrum measurements scan the frequency range 1.78-5.62 kHz in 8 frequency channels. Plasma measurements were acquired with a low-energy proton-electron differential energy analyzer (Lepeudea) similar to previous instruments of this type flown on the Imp and Injun satellites [Frank, 1967]. The Lepeudea instrument on board Hawkeye 1 provides measurements of the directional differential intensities of protons and electrons within the energy range 50 eV $\leq E \leq 40$ keV. A collimated, thin-windowed Geiger-Mueller tube which responds primarily to electrons with $E > 45$ keV over most of the Hawkeye 1 orbit is also included within the Lepeudea instrumentation.

**The Continuum Radiation Event of October 14-21, 1974**

Because of the relatively short electric antenna used on Hawkeye 1, continuum radiation can only be detected when the intensity is well above the normal quiescent level. During the first 6 months of in-flight operation, only one event, from October 14-21, 1974, has been found for which continuum radiation could be detected by Hawkeye 1 during a prolonged period. This event occurred following a magnetic storm which commenced at about 1700 UT on October 14. The hourly equatorial $Dst$ magnetic disturbance index for this event is shown in Figure 2 [Sugiura and Poros, 1974]. Two abrupt depressions are evident in the $Dst$ index near the start of the event: the first at about 0100 UT on October 13 and the second at about 1700 UT on October 14. Although the largest magnetic disturbance is associated with the October 13 storm, both the Hawkeye 1 and the Imp 8 observations show that the onset of the enhanced continuum radiation intensities during this event is associated with the October 14 magnetic disturbance.

The electric field intensities obtained from a series of Hawkeye 1 passes through the magnetosphere during the October 14-21 event are shown in Figures 3 and 4. The intensities are given in eight frequency channels from 13.3 to 178 kHz. The ordinate for each channel is proportional to the logarithm of the electric field strength. The interval from the base line of one channel to the base line of the next higher channel represents a dynamic range of 100 dB. The receiver noise level is located slightly above the base line of each channel. With the exception of orbit 67 these passes are all for the inbound portion of the orbit in the local morning, at about 0600 $\pm$ 1.0 hours magnetic local time. Usually, the continuum radiation is completely masked by intense auroral kilometric radiation on the outbound passes in the local evening. The outbound pass on orbit 67 is shown because no observations are available for the immediately preceding inbound pass. The start times are

![Hawkeye Orbit Number Graph](image)

**Fig. 2** The geomagnetic $Dst$ index during the October 14-21, 1974, continuum radiation event. The times for the Hawkeye 1 orbits analyzed in this paper are shown at the top of this plot.
adjusted so that the perigee is located at the right-hand edge of each plot, except for orbit 67, where perigee is on the left. The spacecraft coordinates are therefore similar for each plot. The universal time (UT) and geocentric radial distance to the spacecraft, in earth radii ($R_E$), are given at the bottom of each plot. The radial distance in these graphs varies from about 9.0 $R_E$ at $60^\circ$ magnetic latitude over the northern polar region to about 1.4 $R_E$ at perigee over the southern polar cap.

The first pass shown in Figure 3, on October 14 (orbit 63), occurs shortly before the onset of the October 14 magnetic storm. The electric field noise levels are very low throughout this entire pass, the only significant noise being the $(n + 1)f_a$ noise bands of the type described by Shaw and Burnett [1975] near the plasmapause. A representative power spectrum is shown in the top panel of Figure 5 at a radial distance of 5.93 $R_E$ for this pass. Except for a very small enhancement at 31.1 kHz no continuum radiation is detectable. The second pass shown in Figure 3, orbit 63, occurs approximately 2 days later, on October 16. On this pass, several types of radio emissions are present. At high frequencies, in the 56.2- to 178-kHz channels, moderately intense auroral kilometric radiation is evident during the first part of the pass, before about 1200 UT. At somewhat lower frequencies, from about 23.7 to 100 kHz, an almost constant amplitude broad band noise is present slightly above the receiver noise level. Even though this noise is very weak, it is easily distinguished from the receiver noise level by the spin modulation caused by the antenna rotation. The period of this modulation is about 2 min, which is the beat period between the spacecraft rotation rate and the telemetry sampling rate.

A representative spectrum of the broad band noise observed on orbit 63 is shown at a radial distance of 5.63 $R_E$ in the center panel of Figure 5. This spectrum is qualitatively similar to the continuum radiation spectrums observed by the Imp 6 and Imp 8 satellites at $\sim 30 R_E$. However, the intensities are about 20 dB greater than the continuum radiation intensities normally observed by Imp 6 and Imp 8 during quiet times, even after correcting for the expected $1/R^2$ variation with radial distance. These high intensities are consistent with the continuum radiation intensities reported by Franke [1973] for geomagnetically disturbed periods ($Kp > 5$). On the basis of these comparisons we identify the broad band noise observed
Continuum radiation of comparable intensity is observed again 2 days later, on October 18 (orbit 64), and again 4 days later, on October 20 (orbit 65). However, 6 days later, on October 22 (orbit 66), the intensity has decreased to essentially undetectable levels at all frequencies above about 30 kHz. A power spectrum for the inbound pass of October 22 is shown in the bottom panel of Figure 5 at a radial distance of 7.16 \( R_E \). No telemetry was received after about 2110 UT on this pass, so measurements are not available at radial distances less than about 7.0 \( R_E \). During the immediately following outbound pass of October 23 (orbit 67 in Figure 4), only a few hours later, no continuum radiation is detectable at any radial distance.

The enhanced continuum radiation during this event was also detected by the University of Iowa plasma wave experiment on the Imp 8 spacecraft (for details of this experiment, see Gurnett [1975]). During the first few days of the event, Imp 8 was at a radial distance of about 28–46 \( R_E \) on the day side of the earth and in an excellent position to monitor the continuum radiation without excessive interference from the night side auroral kilometric radiation. The radiation intensities detected by Imp 8 at frequencies from 31.1 to 178 kHz are shown in Figure 6 for this period. Since the continuum radiation intensities are only slightly above the receiver noise level, it is difficult to clearly identify the continuum radiation at all times during the event. The onset of the enhanced continuum radiation is, however, clearly evident at about 1700 UT on October 14, essentially coincident with the onset of the October 14 magnetic disturbance (see Figure 2). At 56.2 kHz the intensity increases about 20 dB above the normal quiescent level present before the onset of the event. The rise to maximum intensity takes place in approximately 1 hour, from 1700 to 1800 UT. The enhanced continuum radiation remains easily detectable for at least 3 days, through October 17. During this period the intensity varies by as much as 10 dB, typically on a time scale of a few hours. After October 17 the continuum radiation is almost completely masked by intense auroral kilometric radiation as the spacecraft progresses into the local evening region.

The spin modulation of the continuum radiation observed by Imp 8 during this event has been analyzed to determine the
location of the source of this noise. Details of the analysis procedure are given by Kurth et al. [1975]. Figure 7 shows the direction-finding measurements obtained from Imp 8 for the continuum radiation observed at 56.2 kHz during the October 14-21 event. The spacecraft position for each measurement is shown as a dot, and the line extended from each dot indicates the direction of the source projected into the ecliptic ($X_{ecl}$, $Y_{ecl}$) plane. Since the rotation axis of the Imp 8 electric antenna is directed perpendicular to the ecliptic plane, it is only possible to determine the direction to the source projected onto the ecliptic plane. Only certain selected periods are analyzed because of interference from other types of noise. Each direction-finding measurement in Figure 7 represents a 1- to 6-hour integration interval.

By assuming that the source distribution remained constant during the event the apparent center of the source region can be estimated from the intersection of the ray paths observed at various local times. It is evident from Figure 7 that most of the continuum radiation during this event appears to come from the dawn side of the magnetosphere at a radial distance, projected onto the ecliptic plane, of about 2-5 $R_E$ from the center of the earth. Modulation index measurements also show that

![Graphs showing power flux before, during, and after the continuum radiation event.](image)

Fig. 5. Selected spectrums of the received power flux before, during, and after the continuum radiation event at comparable radial distances from the earth. All three spectrums are from inbound passes in the local morning.
the source is very broad, several earth radii in diameter. These results are similar to previous direct-find measurements of continuum radiation during less disturbed periods [Gurnett, 1975].

**Outer Zone Electron Intensities During the Event of October 14–21, 1974.**

To try to identify the charged particles responsible for the enhanced continuum radiation during this event, we have analyzed observations of the electron and proton intensities for each pass of Hawkeye 1 through the outer radiation zone during the October 14–21 event.

Plates 1 and 2 summarize the electron energy spectra obtained with the Lepede on Hawkeye 1 before, during, and after the period of enhanced continuum radiation. These measurements were made with the ramp operational mode (high temporal resolution) of the Lepede and cover the energy range 70 eV to 27 keV. Each spectrogram shows the responses of the electron electrostatic analyzer as a function of energy and time for a 1-hour period as the spacecraft passes through the outer radiation zone at $L$ values ranging from about 4 to 10. The ordinate of each spectrogram gives the electron energy on an approximately logarithmic scale from 70 eV to 27 keV. The analyzer responses are color coded according to the logarithmic scale on the right-hand sides of these figures, the high responses being red and the lowest responses being blue. For these series of spectrograms the satellite spin axis is aligned approximately parallel to the local magnetic field direction. Since the axes of the fields of view of the Lepede are directed perpendicular to the spin axis, the pitch angles sampled by the instrument are ~90°. The starting times of the spectrograms are chosen to provide a set of measurements corresponding to approximately the same range of $L$ values for each pass. The spectrograms of Plate 1 are for inbound passes in the local morning sector of the magnetosphere, and the spectrograms of Plate 2 are for outbound passes during local evening. Because of telemetry coverage limitations, observations are usually not available for consecutive inbound and outbound passes through the outer radiation zone. To study temporal variations over a several-day period, it is therefore necessary to intercompare measurements on the inbound (local morning) and outbound (local evening) passes.

The inbound pass on October 14 (orbit 62) in Plate 1 occurs approximately 7 hours before the onset of the enhanced continuum radiation as determined by Imp 8. The electron in-
tensions during this pass are typical of quiet conditions in the outer radiation zone in the local morning. Moderate intensities of low-energy, ~1 keV, electrons are present at large $L$ values, $L \gtrsim 8$, but the intensities of these electrons decrease rapidly with decreasing $L$. These electrons are believed to be injected from the night side plasma sheet and are convected through the dawn and dusk side magnetosphere by the combined effect of gradient drift and the dawn-dusk electric field. The quiescent electron intensities observed during this pass show that the magnetic disturbance on October 13 did not produce a durable injection of low-energy electrons into the outer radiation zone.

The next inbound pass occurs on October 16 (orbit 63), approximately 2 days after the onset of the enhanced continuum radiation. The outer zone electron intensities encountered during this pass have changed radically since the preceding pass. A very substantial increase of electron intensities within the energy range from ~1 to 30 keV is evident deep within the magnetosphere at $L$ values from about 4 to 8. The energy spectrums of these electron intensities are very broad and exhibit peak differential intensities at energies from about 1 to 10 keV. Representative electron energy spectrums at $L \approx 5.8$ for the inbound passes of orbit 62 (prior to electron injection) and orbit 63 (first pass after injection) are compared in Figure 8. Prestorm electron intensities are similar to those reported by Lyons and Williams [1975] at these $L$ values near the magnetic equator. There is some evidence of satellite charging at these $L$ values when the spin modulation of electron intensities at $E \lesssim 400$ eV is examined, a topic which will be discussed more thoroughly in a future paper (see, for example, the time period 1240–1300 UT of the spectrometer for orbit 63 of plate 1). Specifically, the electron spectrums at these lower electron energies exhibit peak intensities only once per satellite spin period, these maxima being closely centered at the minimum solar aspect angle relative to the instrument fields of view. The Lepedea is sampling pitch angles at about 90° for the entire spin period. Hence only differential electron intensities at $E > 900$ eV have been shown in Figure 8. The electron spectrums at $E > 900$ eV are not significantly affected by these charging events. The electron intensities on orbit 63 are among the highest encountered to this date for this region of the outer zone.

Subsequent passes on October 18 (inbound orbit 64 and outbound orbit 65) and on October 20 (inbound orbit 65 and outbound orbit 66) continue to reveal the presence of this energetic electron distribution deep within the outer zone. During the 4-day period from October 16 to October 20 the intensity of these electrons gradually decreases, and the spatial distribution slowly spreads over a broader range of $L$ values. A striking dawn-dusk asymmetry in electron intensities is also evident upon examination of Plate 1 (local morning) and Plate 2 (local evening). By comparing the inbound and outbound passes on October 18 and October 20 one can see that both the intensities and the average energies are greater in the local morning than in the local evening. Electron intensities are also found on significantly lower $L$ shells during local morning. By October 23 (orbit 67) the electron intensities for $5 \lesssim L \lesssim 8$ have decreased to about the prestorm values.

These observations show that extremely high electron intensities were injected deep into the outer radiation zone sometime between approximately 0930 UT on October 14 and 1245 UT on October 16. Most likely this injection took place during the magnetic storm on October 14. After this injection the electron intensities slowly decreased over a several-day period, through October 20, and subsided to prestorm intensities by October 23.

**DISCUSSION**

We have shown that unusually intense fluxes of low-energy, 1 to 30 keV, electrons were observed deep in the outer radi-

![Fig. 7. A series of direction-finding measurements obtained from the IMP 8 spacecraft at various times during the October 14-21 event. The continuum radiation appears to be coming from a broad region centered in the dawn sector of the magnetosphere about 2-5 $R_e$ from the center of the earth.](image-url)
After this magnetic disturbance, both the enhanced continuum radiation and the intense low-energy electron fluxes were observed through October 20, with a qualitatively similar rate of decay of the respective intensities. Direction-finding measurements showed that the continuum radiation originated primarily from the morning side of the magnetosphere. The low-energy electron intensities also showed a strong dawn-dusk asymmetry, the largest intensities occurring on the morning side of the magnetosphere. These observations all suggest that the enhanced continuum radiation is associated with the intense fluxes of low-energy electrons injected into the magnetosphere during this event.

Although the enhanced continuum radiation occurs during the same interval in which the intense fluxes of low-energy electrons were detected, it is still not immediately certain just what range of electron energies is primarily responsible for the continuum radiation or just what mechanism is involved in the generation of the noise. If the radiation is produced by the incoherent gyrosynchrotron process, as was proposed by Frankel [1973], then most of the radiation must come from high-energy, 200 keV to 1 MeV, electrons, since the mechanism requires relativistic energies to produce significant power fluxes. However, if the radiation is associated with lower electron energies, 10 keV or less, then a coherent plasma instability is required to explain the observed intensities. Because HAWKEYE 1 does not provide measurements of the electron energies involved in the gyrosynchrotron process and since we know of no other spacecraft which can provide suitable measurements during this event, the gyrosynchrotron mechanism cannot be tested directly.

Although HAWKEYE 1 does not have adequate measurements of the high-energy electron intensities to test the gyrosynchrotron mechanism directly, several factors suggest that a coherent plasma instability, involving interactions with low energy electrons, is responsible for the observed continuum radiation.
energies, ~10 keV or less, should be seriously considered to explain continuum radiation events of this type. First, the temporal variation of the continuum radiation intensity during the storm corresponds closely with the observed temporal variation of the low-energy, ~10 keV, electron intensities but not with the expected temporal variation of the high-energy, ~500 keV, electron intensities. After the onset of the October 14 magnetic storm the intensity of the continuum radiation increased almost immediately, within a few hours or less. This time scale is comparable with the time scale in which electrons with energies of ~10 keV are injected deeply into the outer zone during a magnetic storm. The intensities of electrons with higher energies, ~500 keV, however, usually decrease dramatically during the initial phase of a magnetic disturbance on these L shells and then gradually increase to maximum intensity several days after the onset of the storm [Owens and Frank, 1968]. Since the primary electron injection into the outer radiation zone during this event probably took place in association with the October 14 magnetic storm, enhanced ~500 keV intensities are not expected until several days after the onset of the continuum radiation. Second, from examination of the radial variation of the electric field intensities, such as is seen in Figures 3 and 4, it often appears that the continuum radiation is closely associated with intense bands of electrostatic noise which occur just outside the plasmapause. Two excellent examples of relationships of this type are shown in Figure 13 of Gurnett [1975]. Our Figure 4 shows a similar case in which intense bands of electrostatic noise are evident near the low-altitude cutoff of the continuum radiation at f = f_p (from about 1945–2000 UT in the 42.2-, 56.2-, and 100.0-kHz channels). These electrostatic noise bands occur at high-order (n + 1)/f_p harmonics of the electron gyrofrequency f_g and are particularly intense at frequencies near the electron plasma frequency f_p [Shaw and Gurnett, 1975]. The relationship between the continuum radiation and the (n + 1)/f_g noise bands suggested by these observations is illustrated in Figure 9, which shows an idealized radial profile of the frequency spectrums for both types of noise. Usually, the electrostatic noise bands at (n + 1)/f_g = f_p are most intense near and immediately outside of the plasmapause, although in some cases these bands extend almost to the magnetopause. Third, as was discussed by Frankel [1973], the power radiated by the incoherent gyrosynchrotron mechanism is about a factor of 5–15 too small to explain the observed continuum radiation intensity for a typical distribution of high-energy electrons injected into the outer radiation zone during a magnetic storm. A coherent radiation process could easily account for the power flux of the continuum radiation.

Several processes occur by which coherent plasma waves could produce electromagnetic emissions such as the continuum radiation. If the electric field amplitude of the plasma waves is sufficiently large, nonlinear interactions can cause the generation of electromagnetic radiation. Nonlinear interactions of this type are thought to be the mechanism by which type 3 radio noise bursts are produced from electrostatic plasma oscillations [Ginzburg and Zheleznyakov, 1958]. Gurnett and Shaw [1973] proposed that the trapped continuum radiation (referred to as f > f_p electromagnetic noise) is produced by coherent cyclotron radiation from the high-order, n/f_g, rotating charge distributions associated with the electrostatic (n + 1)/f_g electrostatic noise bands. Essentially, the electrostatic wave acts to organize the phases of the electrons, thereby greatly increasing the power radiated at the high harmonics of the cyclotron frequency. Scarf [1974], using a similar mechanism, proposed that wave-wave coupling between the (n + 1)/f_g electrostatic noise bands is involved in the generation of decametric radio emissions from Jupiter. These various mechanisms suggest that it is entirely possible, on theoretical grounds, that the enhanced continuum radiation is generated by low-energy, 1 to 30 keV, electrons via interactions with electrostatic plasma waves. Further investigation of similar events with instrumentation suitable for directly testing the gyrosynchrotron mechanism is needed to definitely establish the mechanism by which this radiation is produced.

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References


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