The POLAR Code Wake Model: Comparison With in Situ Observations

G. Murphy
Jet Propulsion Laboratory, Pasadena, California

I. Katz
S-Cubed, La Jolla, California

Measurements of the ion and electron densities associated with the wake of the shuttle orbiter were made by the Plasma Diagnostics Package (PDP) during the 1985 Spacelab 2 mission. Cross sections of the wake at distances of 30-250 m downstream and measurements along the wake axis from 5 to 100 m were obtained. The POLAR wake model, developed for The Air Force Geophysics Laboratory to study charging of spacecraft in low-altitude high-inclination orbits, was used to perform a three-dimensional simulation of the plasma wake evaluated at points along relative trajectory of the PDP. The POLAR code uses several simplifying assumptions to predict wake densities. These include neglecting the magnetic field and assuming that the plasma is quasi-neutral. The code models plasma density ahead of the expansion front, using a neutral approximation, and models the plasma density behind the expansion front by using the self-similar solution of the expansion of a plasma into a vacuum. For cases where \( T_i \approx T_e \), the front is not sharp and thermal motion can account for most of the expansion. This approach is computationally very efficient. The results presented here are the first known comparison between such a model and actual in situ data obtained for objects of scale size \( \sim 10^4 \lambda_d \). Excellent qualitative and quantitative agreement are found at distances greater than \( \approx 30 \) m, indicating that at least to first order, the model's approximations are justified. An intriguing disparity about the accuracy of the POLAR model to conclusions it contains. Several other investigations have studied the applicability of the self-similar mathematics [Samir et al., 1983; Raychaudhuri et al., 1986; Gurevich et al., 1969; Diebold et al., 1987; Kozima et al., 1988] to wakes. It is our purpose only to determine if POLAR provides a reasonable model for the wakes of "large" objects in the ionosphere, as it has for \( T_i = T_e \) plasmas in the laboratory.

We describe briefly the POLAR model and review the physics it contains, compare the data with the model, and then discuss the range of validity of the code.

The Boltzman relation

\[
    n_e = n_0 \exp \left( \frac{e \phi}{kT_e} \right)
\]  

(1)

Copyright 1989 by the American Geophysical Union.
Fig. 1. The POLAR wake code distinguishes three regions of interest. The ambient plasma, the region of self-similar model, and the neutral approximation spaces are bounded by the Mach cone \( Z = -S_0 t \) and ion front, respectively. The coordinate system used is consistent with equations (1) through (10).

Continuity
\[
\frac{\partial n}{\partial t} + \frac{\partial (n v)}{\partial z} = 0
\]

Equation of motion
\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{e \mathbf{E}}{M \mathbf{v}}
\]

Poisson's equation
\[
\frac{\partial^2 \phi}{\partial z^2} = 4 \pi n (e_e - e_i)
\]

where
- \( n_0 \) ambient density;
- \( n_i \) ion density;
- \( n_e \) electron density;
- \( T_e \) electron temperature;
- \( e \) electron charge;
- \( \phi \) local potential;
- \( k \) Boltzman's constant.

and where \( z \) is a variable representing distance parallel to the front velocity or, in this case, perpendicular to the orbital velocity.

Crow et al. [1975] have numerically solved (1) through (4) to predict the position of the ion front. Katz et al. [1985] developed an analytical fit to the Crow results:

\[
Z_P(t) = 2 \lambda_d \left( \omega + \frac{1}{\alpha} \right) \ln \left( 1 + \omega \alpha t \right) - \omega \alpha t
\]

\[- \left( 1 - \frac{0.429}{\alpha} \right) \left( \omega + \frac{1}{\alpha} \ln \left( 1 + \omega \alpha t \right) \right) \]

where
- \( \omega \) is the ion plasma frequency and \( \lambda_d \) is the Debye length, respectively, and \( \alpha \) is a free parameter determined to be \( \approx 1.6 \).

Katz et al. [1985] showed that this formula agrees well with laboratory data from Wright et al. [1985] and incorporated it in POLAR. Ahead of this front \( Z_P \), the plasma is assumed to expand owing to thermal motion, the so-called "neutral approximation." Behind \( Z_P \) the plasma evolves into a state which is self-similar [Chan et al., 1984]. The self-similar solution of (1)-(4) for \( z > -S_0 t \) is

\[
n = n_0 \exp \left[ \frac{(z + S_0 t)}{S_0 t} \right]
\]

where \( S_0 = (kT_e/M)^{1/2} \) is the ion acoustic speed.

The time variable is defined as

\[
t = \frac{x}{V_0}
\]

where \( x \) is the distance behind the object (perpendicular to \( z \)) and \( V_0 \) is the orbital velocity. We define the self-similar variable \( \xi \) as

\[
\xi = \frac{z}{S_0 t}
\]

Thus the self-similar solution essentially states that between the region bounded in positive \( z \) by the front \( Z_P \) and in negative \( z \) by the line \( z = -S_0 t \), the density rises exponentially to be equal to the ambient value along \( z = -S_0 t \). This is an intuitively reasonable result.

In summary, the wake routines in POLAR employ two limiting cases. (1) Ahead of the ion front the electric field is negligible and the motion of ions is identical to neutrals. (2) Behind the ion front, whose position is determined by (5), the quasi-neutral self-similar solution of (6) is implemented. POLAR has routines which model accurately the geometry of the object, and the "neutral ion" trajectories are calculated from

\[
f(t, x, v) = g(x, \Omega) f_0(v)
\]

where \( f_0(v) \) is the unperturbed distribution function for a drifting Maxwellian, and \( g(x, \Omega) \) has value "0" if a ray starting from \( x \) and going in the direction \( \Omega \) would strike the vehicle and "1" if it would not.
The local density is given by

\[ n_i(x) = \int f_i(x, v) = \int g(x, \Omega) \left( \int f_d(v, \Omega) v^2 dv \right) d\Omega \quad (10) \]

This initial density calculated in three dimensions for neutral particles is compared with density calculated assuming the complex geometric object is replaced by a flat plate at a position where the dominant source appears at the object edge. This ratio provides a "geometric correction factor," which is applied to the quasi-neutral one-dimensional solution discussed earlier for positions behind \( Z_F \). In this way, POLAR can calculate quite rapidly an approximate value for the ion and electron densities in the wakes of complex objects.

Note that the assumptions behind the front are (1) that the electron temperature and ion mass govern the equation of motion, (2) that the plasma is quasi-neutral, (3) that the magnetic field does not affect the ion or electron motion, (4) that equation (5) serves as a good approximation for determining the boundary of the ion front, and (5) that the geometric correction factor calculated in detail with the three-dimensional neutral model can be approximately applied to correct the plasma densities as well. Therefore the algorithm can address complex geometries but takes advantage of the smooth wake structure characteristic of ionospheric plasmas where \( T_e/T_i \approx 1 \). Additionally, the model ignores fields existing in a sheath near the body surface, which should not be of concern in our case where the orbiter is near plasma potential. This implies that ion acceleration calculated by POLAR is dominated by electric fields due to space charge separation in the wake.

**Comparison of Results**

Figure 2 is a plot of the normalized density as a function of time for the entire free-flight time period. Areas of interest are labeled in the figure. The assumed normalization values for the plasma are \( n_e = 1 \times 10^5 \text{ cm}^{-3} \) and \( T_e = 2500\text{ K} \).

The density ratio \( N_{wake}/N_{ambient} \) varies from approximately 0.9 for wake transit (WT) 1, which is at a distance of 245 m, to 0.2 for the closest, WT 3, at 45 m.

One must be careful when comparing the model to the data, since the model assumes a fixed background density of \( 10^5 \text{ cm}^{-3} \), whereas the actual ionospheric density can and does vary considerably. The background density chosen for the model is typical of that observed in the dayside ionosphere. The assumed model temperature is also a constant 2500K. The observed temperature varies plus or minus \( \pm 25\% \) from this assumed value during times of interest. The data are near plasma potential. This implies that ion acceleration observed during the back-away maneuver, with a constant \( 10^6 \text{ cm}^{-3} \) as the reference density [Murphy et al., 1989]. The data are normalized to a constant value, since it is clear some variation in background density occurs but it is not clear exactly what that variation is. To normalize to any unknown value other than a constant would introduce artificial variation and skew the results. Murphy et al. [1989] discuss this issue in detail and in that case, WT 3 is normalized by the data from the prior orbit. However, Murphy et al. [1989] are attempting no comparison to a model. During this wake transit the background is believed to vary by as much as 10% between the start of the observation and the end of the observation.
Fig. 3. Ion and electron data during the back-away are plotted in Figure 3a, normalized to \( n_e = 1 \times 10^6 \text{ cm}^{-3} \). Note the relatively good agreement between model and data between 30 and 75 m. Beyond \( t = 20 \text{ min} \) (75 m), background density varies considerably, as is illustrated in Figure 3b, which includes data for one orbit later than Figure 3a, at approximately the same station-keeping position, superimposed on a predicted density for this orbit from the IRI ionosphere model.

[Murphy et al., 1989] so the model cannot be tested to an accuracy greater than that.

**DISCUSSION**

In examining the back-away density profile, we find three relevant observations from Figure 3.

1. Close to the orbiter (<30 m) the model underestimates the observed density by 1 to 2 orders of magnitude.

2. In the range 30–75 m the model predicts quite accurately the gradual increase in density until the time \( t = 20 \text{ min} \) (Figure 3). After \( t = 20 \text{ min} \) the observed density seems to have a variation, which is not believed to be wake-related. These density changes result from ionospheric variability as the spacecraft approaches the dawn-dusk meridian plane and are predicted by empirical models such as IRI.

Considering the first observation, recall that the assumptions incorporated within the POLAR wake model require a quasi-neutral plasma, assume a self-similar solution, and neglect magnetic fields. Since the electron and ion densities observed even at the beginning of the release and back-away period agree within 10%, it would seem that quasi-neutrality would be valid. It has been shown by Chan et al. [1984] that after a few ion plasma periods (=0.1 ms in this case) the plasma expansion becomes self-similar. For the case of the shuttle orbiter, this takes place within the first \( \approx 1 \text{ m} \) of the wake. The magnetic field, if it is to be considered for this

**TABLE 1. Comparison of Observations From Murphy et al. [1989] With POLAR Predictions**

<table>
<thead>
<tr>
<th>Wake</th>
<th>Latitude, deg</th>
<th>( T_e, \text{ K} )</th>
<th>Distance, m</th>
<th>POLAR Normalized Density</th>
<th>Observed Normalized Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-35</td>
<td>2500</td>
<td>242</td>
<td>0.9</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>-45</td>
<td>3000</td>
<td>125</td>
<td>0.75</td>
<td>0.61</td>
</tr>
<tr>
<td>3</td>
<td>+10</td>
<td>1500</td>
<td>45</td>
<td>0.20</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>+35</td>
<td>2000</td>
<td>105</td>
<td>0.65</td>
<td>0.90</td>
</tr>
</tbody>
</table>
case, would always act to limit plasma flow, rather than enhance it. Therefore it too can be eliminated from serving as an explanation for the poor fit at less than 30 m.

The answer to the disparity between model and observation would seem to lie in the role played by contaminant ions. Murphy et al. [1989] discuss these ions and offers their presence as an explanation for the disparity between these data and that taken at similar distances while on the remote manipulator system (RMS).

Let us turn now to the 30- to 75-m distance region. The most dominant characteristic of both the data and model is the relatively smooth increase in density as the PDP moves axially along the orbiter wake.

This midwake region has been studied extensively in the laboratory and the wake-fill process depends strongly on the body size, body potential, and ratio of ambient ion to electron temperature. Stone [1981], Fournier and Pigache [1975], Hester and Sonin [1970], and many others have performed laboratory experiments and observed fine structure in wakes, including ion density peaks along the wake axis and wavelike condensation disturbances. It is important, however, to note that for the case of large bodies in low earth orbit (LEO) (1) the body potential is not too different from the plasma potential (a few \( kT_e \) at most, since the body surface is an insulator and does not expose \( \mathbf{v} \times \mathbf{B} \) potential to the plasma); (2) the plasma is a cold Maxwellian (\( kT \)) and collisionless; and (3) the ambient ion and electron temperatures are close to being equal.

An excellent review of laboratory work before 1975 is given by Fournier and Pigache [1975]. Another excellent review of the subject of expansion of plasma into the wake is given by Samir et al. [1983]. In these cases the authors agree with the basic finding of Gurevich and Pitaevskii [1969] that the fine structure and ion peaks observed in certain laboratory investigations vanish as \( T_i \) approaches \( T_e \). We see in this case that in spite of (or perhaps because of) the effect of contaminant ions, we have a large-scale wake which is basically devoid of any fine structure, at least in the sense of total electron or ion density. It should be emphatically noted, however, that this does not imply knowing all there is to know about the wake structure. The overall plasma density is only the zeroth order parameter. Ion and electron composition [Grebowsky et al., 1987]; vector measurements of ion velocities [Stone et al., 1983, 1988]; electron temperature [Murphy et al., 1986; Raitt et al., 1984]; plasma turbulence [Raitt et al., 1984] all play roles in understanding the total physics of the wake structure for such a complex, gas-emitting, large object.

As discussed earlier, the structural differences between predictions and observations after \( t \approx 20 \) min in Figure 3a are attributed to natural ionospheric variation (not modeled by POLAR). Figure 3b illustrates the density profile one orbit after the back-away maneuver and shows this similar structure.

Let us compare the model predictions to observations for WT 1 through WT 4. The agreement is quite remarkable and affirms that the "well-behaved" wake structures associated with \( T_i/T_e \approx 1 \) plasmas can be adequately modeled by the physics contained in the POLAR model. There is only one significant difference between model and data. WT 2, which occurs at \( \approx 125 \) m, seems to be considerably deeper than WT 4, which occurs at a little more than 100 m. Murphy et al. [1989] discuss this extensively, and we do not believe that, considering approximations made by the model and errors made in normalization, we could expect any better agreement. If the magnetic field, contaminant ions, and orbiter sheath do play some role, it is clear from both the model and the data that it must be a secondary one.

Studying the detail of WT 3 observations and POLAR's predicted profile, we also find good agreement. This is significant because at 45 m downstream the details of the orbiter geometry and its effect on the wake can not yet be "washed out" (the long dimension of the orbiter is \( \approx 36 \) m). The agreement between model and data seems to imply that the geometric assumptions are valid and that it is permissible to use the geometric correction factor calculated from the neutral flow model, at least to first order.

Note that the center of the predicted wake seems to be
offset slightly from that observed and that the predicted density gradient seems slightly greater than observed. Murphy et al. [1989] discuss the accuracy with which the trajectory reconstruction takes place. This offset error is consistent with that level of precision. It is also possible that errors in normalization (constant background assumed) could produce this effect.

The difference in density gradients may be due to a slightly different plasma temperature than that modeled or may be consistent with the role played by contaminant ions in the neutralization of the space charge electric field.

Electric field data is difficult to discern from the PDP measurements because of interference from another instrument, but J. Steinberg (private communication, March 1988) has examined data from the time period of WT 3 and finds an electric field which changes sign at the wake center. This field is within a factor of 2 of that expected from a self-similar expansion. No attempt has been made to compare the predicted field computed by POLAR to the actual data, since error bars on the data are rather large. The authors may examine this as well as the RMS data [Tribble et al., 1989] in more detail in a future paper. The RMS roll data may be more useful for comparison, since those data are taken at <10 m downstream, where the density depletion and electric fields are greater.

Conclusions

The sampling of PDP ion and electron densities verifies that for ionospheric plasma conditions ($T_i \approx T_e$), the orbiter wake is relatively smooth in its structure from =30 m to distances of =250 m downstream. POLAR predicts this smooth wake structure and agrees with the observations to an accuracy of <30% (Table 1). For large and complex systems such as the orbiter, outgassed products may play a significant role in the structure of its wake at distances less than the characteristic body dimension. Adequate modeling in this regime requires input that details the outgassed species and rates, chemistry describing their interaction with the ionosphere, and inclusion of magnetic field to account for the pick-up ion population in the wake.

At midwake distances the magnetic field effects, if any, would appear to be secondary to the dominant role of the electric field and thermal motion already modeled by POLAR in the wake-fill process. However, for slightly larger objects or high-inclination orbits the magnetic field may have to be considered if accurate results are required. In addition, the fundamental scientific questions associated with large-body wakes will eventually require its inclusion.

For comparisons between in situ observations and models such as POLAR to be meaningful at levels better than a few tens of percent, more than simple axial or planar profiles of density are needed. Future experiments will also require good background measurements and inclusion of vector ion velocity, electric field, and particle distribution functions.

Acknowledgments. The authors wish to express their thanks to the reviewers for their constructive comments and suggestions. This work was supported by NASA MSFC contract NAS8-32807, NASA LeRC grant NAG 3-449, and by the Air Force Geophysics lab under contract F19628.86.C-0056. The Editor thanks W. J. Raitt and two other referees for their assistance in evaluating this paper.

References


I. Katz, S-Cubed, P.O. Box 1620, La Jolla, CA 92038.

G. Murphy, Jet Propulsion Laboratory, MS 301-460, 4800 Oak Grove Drive, Pasadena, CA 91109.

(Received December 23, 1987; revised March 20, 1989; accepted March 22, 1989.)