Electrostatic Turbulence in the Earth’s Central Plasma Sheet Produced by Multiple-Ring Ion Distributions

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A mechanism to generate a broad spectrum of electrostatic turbulence in the quiet time central plasma sheet (CPS) plasma is discussed. We show theoretically that multiple-ring ion distributions can generate short-wavelength (kλD_e ≈ 1), electrostatic turbulence with frequencies ω ≈ kV_j, where V_j is the velocity of the jth ring. Using a set of parameters from measurements made in the CPS, we find that electrostatic turbulence can be generated with wavenumbers 10^{-2} ≤ kλD_e ≤ 1.0, and with real frequencies in the range 0 < ω_i/ω_{pi} < 10, and linear growth rates γ/ω_{pi} > 0.01 over a broad range of angles relative to the magnetic field (5° < θ < 90°). The fastest growing modes are caused by an ion ring drift instability and occur for nearly parallel propagating modes (5° < θ < 10°) with ω_i ≈ ω_{pi} at wavelengths comparable to the electron Debye length. We also find that lower hybrid waves and electron cyclotron waves can be excited for nearly perpendicular propagating modes (θ = 90°). We compare these theoretical results with wave data from ISEE 1 using an ion distribution function exhibiting multiple-ring structures observed at the same time. The theoretical results in the linear regime are found to be consistent with the wave data. Finally, we discuss these results in relation to nonlinear particle dynamics in the CPS which can generate the multiple-ring ion distributions.

1. INTRODUCTION

Recently, Chen et al. [1990] presented observational evidence to support their prediction that nonlinear particle dynamics in the Earth’s central plasma sheet (CPS) can result in quiet time ion distribution functions having resonance structures. The development of these structures is based on the process of “differential memory” [Chen and Palmadesso, 1986] and was first predicted by Burkhart and Chen [1991] as a consequence of the nonlinear particle dynamics in the magnetotail. In essence, differential memory is a process whereby non-Maxwellian distribution functions are formed because of phase space partitioning; these distributions may then drive plasma instabilities. Burkhart and Chen [1991] model the CPS magnetic field as B(x, y) = B_0 tanh (z/8) + B_0z, and they calculate the time evolution of the distribution function f(v, t) in the equatorial plane (z = 0) for t > 0 following a change in source plasma at t = 0 (the source plasma is defined to be in the uniform field region |z| > 48). The results indicate that the distribution function develops local maxima and minima with their locations scaling as θ^{1/4}, where θ = (mv^2/2)(mb^2Ω^2Ω_e^2)^{-1} with b_n = B_n/B_0 and Ω_n = eB_n/mc. Huang et al. [1989] and Chen et al. [1990] have presented ISEE 1 data of quiet time ion velocity distribution functions in the CPS which clearly show structures that scale as θ^{1/4}. Figure 1 shows an ion distribution function obtained by the LEPEDEA instrument exhibiting this scaling property. Similar distribution functions have been published previously by Huang et al. [1989]. The data shown represent f(v⊥, v∥) evaluated for v∥ = 0. Thus the peaks in the distribution function can be interpreted as “ion rings.” The questions we address in this paper are the following: (1) What are the stability properties of these ion distribution functions in the CPS? (2) What are the observational signatures of an instability generated by such a distribution function? (3) Can any waves generated be a component of broadband electrostatic noise observed in the magnetotail?

We find that multiple-ring ion distributions can generate short-wavelength (kλD_e ≈ 1), electrostatic turbulence with frequencies ω ≈ kV_j, where V_j is the velocity of the jth ring. Using a set of parameters from measurements in the central plasma sheet, we find that electrostatic turbulence can be generated with wavenumbers 10^{-2} ≤ kλD_e ≤ 1.0, and with real frequencies in the range 0 < ω_i/ω_{pi} < 10 and growth rates γ/ω_{pi} > 0.01 over a broad range of angles relative to the magnetic field (5° < θ < 90°). The fastest growing modes are caused by an ion ring drift instability and occur for nearly parallel propagating modes (5° < θ < 10°) with ω_i ≈ ω_{pi} at wavelengths comparable to the electron Debye length. We also find that lower hybrid waves and electron cyclotron waves can be excited for nearly perpendicular propagating modes (θ = 90°). We discuss these results with respect to wave observations in the magnetotail (e.g., BEN) and nonlinear particle dynamics.

The organization of the paper is as follows. In section 2 we present the basic assumptions of the theory and the dispersion equation. In section 3 we describe both analytical and numerical results. In section 4 we compare the theoretical calculations with observed quiet time CPS wave data. Finally, in section 5 we summarize our findings.

2. BASIC ASSUMPTIONS AND DISPERSION EQUATION

An extensive amount of work has been done on the stability of plasmas with ion ring distribution functions [Mikhailovskii, 1974; Lee and Birdsay, 1979a, b; Wu et al.,...
ELECTROSTATIC TURBULENCE IN THE CENTRAL PLASMA SHEET

Fig. 1. Velocity distribution functions obtained in the central plasma sheet ($v_i = 0$). The solid curve is for ions (left and bottom axes) and the dashed curve is for electrons (right and top axes). The error bars for ions are similar to those of Huang et al. [1989]: in the low ion velocity regime ($v < 5 \times 10^7$ cm/s) the error bars are comparable in size to the dots, while in the high ion velocity regime ($v > 10^8$ cm/s) the error bars are 2-3 times the size of the dots [from Chen et al., 1990].

1984; Akimoto and Winske, 1985; Akimoto et al., 1985a, b].

Space-based applications have been directed at understanding wave phenomena in the Earth's bow shock, comet-solar wind interactions, and solar flares and during the early stages of ionospheric and magnetospheric chemical releases. In this paper we focus on conditions relevant to the central plasma sheet.

We consider the following equilibrium distribution functions. For the electrons we take

$$F_e(v^2) = n_e (1/v_e^2)^{3/2} \exp \left(-v^2/v_e^2\right)$$

(1)

while for the ions we take

$$F_i = F_{i0} + F_{ij},$$

where

$$F_{i0}(v^2) = \frac{n_{i0} (1/v_i^2)^{3/2}}{2\pi v_J^3} \exp \left(-v^2/v_i^2\right)$$

(2)

$$F_{ij}(v_{ij}) = \frac{n_{ij}}{2\pi v_J} \delta(v_{ij} - v_J) \delta(v_{ij})$$

(3)

and $v_e = (2T_e/m_e)^{1/2}$ is the thermal velocity, $V_J$ is the velocity of the $j$th ion ring, and $j = 1, 2, 3, \cdots$ denotes a particular ion ring. We are simplifying our analysis by choosing (3) to represent the ring ions in order to ascertain the basic stability properties of the plasma. Other ring distribution functions have been considered a Lorentzian function and a Dory-Harris-Guest function (see Akimoto et al. [1985a] for details). Finally, we impose $n_e = n_{i0} + \Sigma n_{ij}$ to ensure charge neutrality.

We consider only electrostatic perturbations. Coupling to electromagnetic components can be neglected when $\omega_{pe}/c^2k^2 << 1$, where $\omega_{pe} = (4\pi n_e e^2/m_e)^{1/2}$ is the electron plasma frequency and $k$ is the wavenumber [Akimoto et al., 1985a, b]. This criterion can be rewritten as $k^2e^2 >> r_e$ or $k\lambda_{de} > v_e/c$, where $\rho_e = v_e/\Omega_e$ is the electron Larmor radius, $\lambda_{de} = v_e/\omega_{pe}$ is the electron Debye length, and $\beta_e = 8\pi n_e T_e/B_0^2$; this condition is well satisfied for the short-wavelength, high-frequency modes discussed in this paper. We note that wave observations in the CPS for the periods reported in this paper do show some magnetic fluctuations in the frequency range 10-100 Hz, which suggests that the low-frequency waves have an electromagnetic component.

The plasma and magnetic fields are assumed to be uniform with $B = B_0z$. Perturbed quantities are assumed to vary as $\exp[-i(k_{\perp}x + k_\|z - \omega t)]$, where $k_{\perp} = k \sin (\theta)$ and $k_\| = k \cos (\theta)$. Finally, we assume the ions to be unmagnetized; this is justified because the growth rate of the waves under study satisfy the condition $\gamma > \Omega_i$. On the other hand, no such assumption is made on the electrons; the electron response is determined by the physical parameters of the system. Specifically, the electrons will behave as unmagnetized particles when $\gamma > \Omega_e$ or $k_\|\rho_e >> 2$ [Akimoto et al., 1985a], and as magnetized particles when $\gamma < \Omega_e$ and $k_\|\rho_e << 2$. We will show that for obliquely propagating waves $(k_\| \neq 0)$ the electrons will be unmagnetized, while for orthogonally propagating waves $(k_\| = 0)$ they will be magnetized. The dispersion equation under these assumptions is given by

$$D = 1 + X_e + X_\| + \Sigma X_{ij} = 0$$

(4a)

where

$$X_e = \frac{2\omega_{pe}^2}{k^2v_e^2} \left(1 + \sum_{q = -\infty}^{\infty} \frac{1}{q} \frac{\Gamma_1(b_\|)\xi_\|Z(\xi_\|)}{\Gamma_0(b_\|)}\right)$$

(4b)

$$X_\| = \frac{2\omega_{pi}^2}{k^2v_i^2} \left(1 + \xi_iZ(\xi_i)\right)$$

(4c)

$$X_{ij} = \frac{\omega_{pi}^2}{(\omega^2 - k_\|^2v_J^2)^{3/2}}$$

(4d)

and $\omega_{pe}^2 = 4\pi n_e e^2/m_e$, $\omega_{pi}^2 = 4\pi n_i e^2/m_i$, $\Gamma_1(x) = I_1(x)$ exp $(-x)$, $I_1(x)$ is the modified Bessel function of order 1, $b_\| = k_\perp\rho_e^2$, $\Gamma_0(q) = (\omega - \Omega_e)/k_\|v_\|$, $\xi = \omega/k_\|v_\|$, $Z$ is the plasma dispersion function, $\alpha_0 = n_{i0}/n_e$, and $\alpha_j = n_{ij}/n_e$. Charge neutrality requires $\alpha_0 + \Sigma \alpha_j = 1$.

3. RESULTS

3.1. Analytical Results

Four wave modes are shown to be unstable based upon (4): a lower hybrid instability [Akimoto et al., 1985a], an ion acoustic instability [Akimoto et al., 1985b], a ring drift mode instability, and an electron cyclotron instability. We discuss these modes separately.

3.1.1. Lower hybrid instability. We assume $k^2\rho_e^2 << 1$, $\omega/k_\|v_\| > 1$, $\omega << \Omega_e$, and $\omega/k_\|v_J > 1$. In this case the electrons are strongly magnetized and the dispersion equation reduces to [Mikhailovskii, 1974]

$$D = 1 + \frac{\omega_{pe}^2}{\Omega_e^2} - \frac{\omega_{pi}^2}{\omega^2} \left(\alpha_0 + \frac{m_i}{m_e} \cos^2 \theta\right) - \Sigma \frac{\omega_{pi}^2}{\omega^2 - k_\|^2v_J^2} \sum_{j} \alpha_j = 0.$$
We further assume $a_0 >> a_j$ and find that a lower hybrid wave exists with a frequency $\omega_0 = \pm \omega_0 (a_0 + (m_e/m_i) \cos^2 \theta)^{1/2}$, where $\omega_0 = \omega_{pi}/(1 + \omega_{pe}^2/\Omega_e^2)^{1/2}$ is the lower hybrid frequency. In addition, a set of ring drift modes exist ($\omega_d = \pm k_\perp V_j$). When $\omega_0 = \omega_d$, a resonance can occur between the lower hybrid wave and the ring drift wave leading to instability. Writing $\omega = \omega_0 + \delta \omega$ with $\delta \omega \ll \omega_0$, it is easily shown that

$$\delta \omega_j = \frac{1}{2} a_j^{2/3} \omega_{pi} \left[ \frac{m_i}{m_e} \cos^2 \theta \right]^{1/10} (\cos \pi/5 + i \sin \pi/5).$$

This result is valid for $\cos (\theta) \leq (m_e/m_i)^{1/2}$ and $k = \omega_0/\sqrt{V_j}$. Thus each ion ring can cause instability at $\omega = \omega_0$ with the growth rate given by (6), but the wavenumber at maximum growth will be different for each ring. This mode propagates mainly perpendicular to the ambient magnetic field.

3.1.2. Ion acoustic instability. We assume the electrons are unmagnetized for this mode; this requires that $\gamma > \Omega_e$ and $k^2 \rho_e^2 > 4$ [Akimoto et al., 1985a]. For conditions prevalent in the CPS, the latter condition is appropriate to "demagnetize" the electrons. We further assume that $\omega < k_\perp v_e$, $\omega_0 > k_\perp V_j$, and $a_j < a_0$. The dispersion equation becomes

$$D = 1 + \frac{2 \omega_{pe}^2}{k^2 V_e^2} - \frac{\alpha_0 \omega_{pe}^2}{\omega^2} - \sum \frac{\alpha_0 \omega_{pe}^2}{(\omega^2 - k^2 V_e^2)^{3/2}} = 0.$$  

In writing (7) we have neglected both electron and ion Landau damping. Ignoring the final term in (7), the real frequency of the wave mode is given by

$$\omega_0 = k C_s \left[ \frac{\alpha_0}{2 + k^2 \lambda_{de}^2} \right]^{1/2},$$

where $\lambda_{de} = V_j/\omega_{pi}$ and $C_s = (2T_e/m_i)^{1/2}$. Assuming $\omega = \omega_0 + \delta \omega$ and $\omega_0 = k_\perp V_j$, one can show that

$$\delta \omega_j = \frac{1}{2} (\alpha/\alpha_0)^{2/3} \omega_0 (\cos \pi/5 + i \sin \pi/5).$$

Instability occurs because of coupling between an ion acoustic mode (8) and a ring drift mode.

It is important to keep in mind that (8) and (9) are based upon the assumptions that $\omega_0 >> k_\perp v_e$ and $\omega_0 = k_\perp V_j$. This requires that $V_j \sin (\theta) >> v_i$ and $T_e/T_i >> \alpha_0/(2 + k^2 \lambda_{de}^2)$. For parameters typical of the CPS, the latter requirement is difficult to satisfy because $T_e/T_i < 1$. Thus we do not anticipate that this ion acoustic instability will be viable in the CPS.

3.1.3. Ion ring drift instability. For this instability we make the same basic assumptions as for the ion acoustic instability, but in addition, we take $\omega^2 >> \omega_{pi}^2$. The dispersion equation then reduces to

$$D = 1 + \frac{2 \omega_{pe}^2}{k^2 V_e^2} - \sum \frac{\alpha_j \omega_{pe}^2}{(\omega^2 - k^2 V_e^2)^{3/2}} = 0.$$  

Assuming $\omega = \omega_0 + \delta \omega$ and $\omega_0 = k_\perp V_j$, we find that

$$\delta \omega_j = \frac{1}{2} a_j^{2/3} \omega_{pi} \left( \frac{\omega_{pi}}{k_\perp V_j} \right)^{1/3} \left( k^2 \lambda_{de}^2 + 2 k^2 \lambda_{de}^2 \right)^{2/3} (\cos \pi/3 + i \sin \pi/3).$$

The growth rate achieves a maximum value at $k \lambda_{de} = 2.45$ for constant $\theta$.

The condition that $\omega^2 >> \omega_{pi}^2$ can be rewritten as $k^2 \lambda_{de}^2 >> C_s^2/V_f^2 \sin^2 \theta$. This condition becomes impossible to achieve for reasonable values of $k$ (i.e., $k \lambda_{de} < 1$) as $\theta -> 0$. In this regime it is more appropriate to assume that $\omega < k_\perp v_i$, and expand the ion $Z$ function using a power series. Neglecting ion Landau damping, one recovers essentially the same growth rate as (11); the only difference is that $2 + k^2 \lambda_{de}^2$ is replaced with $2(1 + \alpha_0 T_e/T_i) + k^2 \lambda_{de}^2$. This is valid for $V_j \sin (\theta) < v_i$ which is easily satisfied as $\theta -> 0$. However, for sufficiently small $\theta$ the approximation $\delta \omega \ll \omega_0$ breaks down and (11) becomes invalid; this point becomes clearer in section 3.2.

3.1.4. Electron cyclotron instability. We assume the electrons to be strongly magnetized for this instability. We require $\gamma > \Omega_e$, $k^2 \rho_e < 4$, and $k^2 \lambda_{de}^2 >> k^2 \rho_e^2$. For conditions prevalent in the CPS, this former condition is easily satisfied for the parameters we are considering, while the latter condition requires perpendicular (or very near-perpendicular) propagation. We also take $k^2 \rho_e^2 >> 1$ and $\omega >> \omega_{pe}$ which are appropriate for the parameters to be considered. Under these assumptions the dispersion equation is given by

$$D = 1 + \frac{2 \omega_{pe}^2}{k^2 V_e^2} \left( 1 - \frac{1}{k_\perp \rho_e \pi^{1/2}} \sum \frac{\omega}{\omega - \omega_{pe}} \right) - \sum \frac{\alpha_0 \omega_{pe}^2}{(\omega^2 - k^2 V_e^2)^{3/2}} = 0.$$  

Ignoring the ring component, one can show that the eigenmodes associated with (12) are electron cyclotron waves,

$$\omega_0 = \frac{\omega_{pe}}{1 - e},$$

where $e = [k \rho_e \pi^{1/2}(1 + k^2 \lambda_{de}^2/2)]^{-1}$ and $e << 1$. We now assume that $\omega = \omega_0 + \delta \omega$ with $\delta \omega \ll \omega_0$ and $\omega_0 = k_\perp V_j$. Thus we are considering the coupling of an electron cyclotron mode to an ion ring drift mode. This places the following condition on the wavenumber: $k_\perp = q \Omega_e/V_j$. It is easily shown that

$$\delta \omega_j = \frac{\omega_{pe}}{2} \left( \frac{\omega}{1 - e} \right)^{1/2} \left( k_\perp \lambda_{de}^2 \right)^{1/2} \left( \frac{\omega_0}{\omega_{pi}} \right)^{1/5} (\cos \pi/3 + i \sin \pi/3).$$

where $\omega_0$ is given by (13). For $k^2 \lambda_{de}^2 << 1$ the growth rate scales as $\gamma \propto q^{-1}$, while for $k^2 \lambda_{de}^2 >> 1$ the growth rate scales as $\gamma \propto q^{-1}$.

3.2. Numerical Results

We solve (4) numerically for the following dimensionless parameters: $\alpha_0 = 0.80$, $\alpha_1 = 0.10$, $\alpha_2 = 0.07$, $\alpha_3 = 0.03$, $V_3/V_e = 0.03$, $V_2/V_e = 0.05$, and $V_1/V_e = 0.07$. $T_e/T_i = 0.25$, and $\omega_{pe}/\Omega_e = 10$. In Figure 2 we plot $\omega/\omega_{pi}$ and $\gamma/\omega_{pi}$ versus $k \lambda_{de}$ for $\theta = \cos^{-1} (m_e/m_i) = 88.66^\circ$. (As an additional point of reference, we point out that $k \rho_e = 1.0$ at
the theory in section 3.1.3. Three important points regarding ring drift instability, although there is a hint of it forming at discrepancy is ion Landau damping by the background ions. with \( \omega_r/k_\perp \approx 3/\nu_{ei} \approx 6.56 \times 10^{-3} \) for ring 3. The primary reason for the discrepancy is ion Landau damping by the background ions. There is no local maxima in growth associated with the lower hybrid instability is in the region \( k \lambda_{de} \approx 0.1 \) (which corresponds to \( k \rho_e \approx 1 \)) and manifests itself as the local maxima in the growth rates associated with rings 2 and 3. From the analysis in section 3.1.1 the local maxima should occur at \( k \lambda_{de} \approx 0.063 \) with \( \omega_{pe}/\omega_{ei} \approx 1.07 \times 10^{-2} \) for ring 2 and \( k \lambda_{de} \approx 0.045 \) with \( \omega_{pe}/\omega_{ei} \approx 7.62 \times 10^{-3} \) for ring 3. We find numerically that the local maxima occur at \( k \lambda_{de} \approx 0.082 \) with \( \omega_{pe}/\omega_{ei} \approx 9.43 \times 10^{-3} \) for ring 2 and at \( k \lambda_{de} \approx 0.053 \) with \( \omega_{pe}/\omega_{ei} \approx 6.56 \times 10^{-3} \) for ring 3. The primary reason for the discrepancy is ion Landau damping by the background ions. There is no local maxima in growth associated with the lower hybrid instability for ring 1 because it overlaps with the ion ring drift instability, although there is a hint of it forming at \( k \lambda_{de} \approx 0.1 \). The ion ring drift instability occurs for \( k \lambda_{de} \approx 0.2 \). The analytical results are in excellent agreement with the exact numerical solutions. The growth rates associated with the different ion rings maximize at \( k \lambda_{de} \approx 2.5 \) in agreement with the theory in section 3.1.3. Three important points regarding the ion ring drift modes are the following. First, the growth rates are reasonably large for each of the rings, i.e., \( \omega_{pe}/\omega_{ei} > 10^{-2} \); for the parameters given in Table 1 this corresponds to growth times \( \tau_g \approx 0.08 \) s. Second, the real frequency varies over a considerable range: \( 0 < \omega_r/\omega_{ei} < 10 \). This corresponds to observable frequencies in the range 0 < \( \omega_r < 2 \) kHz using the parameters in Table 1. And finally, these modes are electrostatic and propagating primarily orthogonal to the ambient magnetic field; the modes have wavelengths in the range 0.4 < \( \lambda < 10 \) km.

In Figure 3 we plot \( \omega_r/\omega_{ei} \) and \( \omega_{pe}/\omega_{ei} \) versus \( \theta = \cos^{-1}(m_e/m_i) \) for \( k \lambda_{de} = 1.0 \). As in Figure 2, the solid, dashed, and dotted curves correspond to the eigenvalues associated with ion rings 1, 2, and 3, respectively. The circles denote the analytic values; the real frequency is given by \( \omega_r = k_\perp V_j = kV_j \sin(\theta) \), while the growth rate is given by (11). As indicated by (11), the growth rate increases as \( \theta \) decreases because \( \gamma \propto \sin(\theta) \). The analytic solution for the growth rate associated with rings 2 and 3 is reasonably good for \( 20^\circ < \theta < 90^\circ \); however, for \( \theta > 20^\circ \) the growth rate is reduced because of ion Landau damping by the background ions. The analytic solution for the growth rate associated with ring 1 is quite good for \( 7^\circ < \theta < 90^\circ \); however, for \( \theta < 7^\circ \) the assumption that \( \omega_i \ll \omega_0 \), which is used to obtain (11), breaks down and the analytical solution is no longer valid. Thus, as \( \theta \to 0^\circ \) the growth rate approaches zero as expected; the perpendicular drift \( V_j \) cannot couple effectively to nearly parallel propagating modes.

In Figure 4 we plot \( \omega_r/\omega_{ei} \) and \( \omega_{pe}/\omega_{ei} \) versus \( k \lambda_{de} \) for \( \theta = 88.66^\circ \) (dashed curve) and 89.99° (solid curve). The parameters are the same as in Figure 2. The unstable modes are generated by ion ring 3 (i.e., \( V_3/v_e = 0.07 \) and \( \alpha_3 = 0.03 \)). The real frequency is essentially the same for both cases. The new feature of this figure is the appearance of enhanced growth at the electron cyclotron frequencies \( \omega_r = -\Omega_e \) and \(-2\Omega_e \) (i.e., \( \omega_r = -4.4 \omega_{pe} \) and \(-8.8 \omega_{pe} \)). The wavenumber and growth rate predicted from the analytical theory (denoted by the circles) are given by \( k \lambda_{de} \approx 1.46 \) and \( \omega_{pe}/\omega_{ei} \approx 0.034 \) for \( q = 1 \) and \( k \lambda_{de} \approx 2.90 \) and \( \omega_{pe}/\omega_{ei} \approx 0.024 \) for \( q = 2 \), which are in very good agreement with the numerical solutions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>Ambient magnetic field ( B_0 )</td>
<td>( 15 \times 10^{-5} ) G</td>
</tr>
<tr>
<td>Electron density ( n_e )</td>
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<tr>
<td>Electron temperature ( T_e )</td>
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<tr>
<td>Ion temperature ( T_i )</td>
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</tr>
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<td>Electron thermal velocity ( v_e )</td>
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<tr>
<td>Ion thermal velocity ( v_i )</td>
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</tr>
<tr>
<td>Ion sound speed ( C_s )</td>
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</tr>
<tr>
<td>Ion ring 1 velocity ( V_1 )</td>
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</tr>
<tr>
<td>Ion ring 2 velocity ( V_2 )</td>
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<tr>
<td>Ion ring 3 velocity ( V_3 )</td>
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</tr>
<tr>
<td>Electron cyclotron frequency ( \Omega_e )</td>
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<tr>
<td>Ion cyclotron frequency ( \Omega_i )</td>
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</tr>
<tr>
<td>Lower hybrid frequency ( \omega_{lh} )</td>
<td>6.2 \times 10^1 rad/s</td>
</tr>
<tr>
<td>Electron plasma frequency ( \omega_{pe} )</td>
<td>8.0 \times 10^4 rad/s</td>
</tr>
<tr>
<td>Ion plasma frequency ( \omega_{pi} )</td>
<td>1.9 \times 10^5 rad/s</td>
</tr>
<tr>
<td>Electron Larmor radius ( \rho_e )</td>
<td>3.6 \times 10^5 cm</td>
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<tr>
<td>Ion Larmor radius ( \rho_i )</td>
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<tr>
<td>Electron inertial length ( c/\omega_{pe} )</td>
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<tr>
<td>Electron Debye length ( \lambda_{de} )</td>
<td>1.2 \times 10^4 cm</td>
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<tr>
<td>Electron beta ( \beta_e )</td>
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</table>

Thermal velocities are defined as \( v_{th} = (2T/m)^{1/2} \).
results. We note that for the parameters considered, the angle of propagation must be \( \cos \theta \ll m_e/m_i \) in order for the electrons to be strongly magnetized. The electron cyclotron waves have maximum growth when \( \theta = 90^\circ \).

4. COMPARISON WITH OBSERVED DATA

During the time period 0217–0241 UT, February 14, 1979, weak electrostatic turbulence was observed in the frequency range 5.6 Hz to 10 kHz. In Figure 5 we show three line spectra plots covering this time period (the distribution function shown in Figure 1 covers the time period 0225–0233 UT). In each panel the top curve is the peak for the 8-min time period, the middle curve is the RMS average, and the bottom (dashed) curve is the instrument noise level. The high levels at the very lowest frequencies are due to solar array noise. The frequency peak at \( f \approx 50 \) kHz is the terrestrial nonthermal continuum radiation and the peak above \( f > 100 \) kHz is the auroral kilometric radiation.

The changes that occur in the line spectra during this time period reflect the dynamic environment of the CPS. In Figure 5a there is a broad peak in the intensity from 100 Hz to \( \sim \) 1 kHz. However, in Figure 5b (8 min later) we find enhanced wave activity and two distinct peaks appear; they occur at \( f \approx 30 \) Hz and at \( f \approx 900 \) Hz. In Figure 5c it is seen that the low-frequency peak at \( f > 30 \) Hz continues to intensify, while the high-frequency peak at \( f > 900 \) Hz broadens and shifts its maximum to a slightly higher frequency \( f \approx 2 \) kHz. We also note that four brief periods of low frequency magnetic bursts have been identified in the time period 0207–0232 UT: 17.8 Hz at 0211 UT, and 17.8 Hz and 31.1 Hz at 0216 UT, 0217–0218 UT, and 0222–0223 UT.

About 10 magnetic bursts are evident between 0232 and 0243 UT in the frequency range 10–100 Hz. They are most intense in the 31.1-Hz and 56.2-Hz channels, especially at 0222–0233 UT, 0240 UT, and 0242–0243 UT. This suggests that at times, the low-frequency waves have an electromagnetic component.

We now compare these observational results with the theoretical analysis presented in section 3. The theoretical results are shown in Figure 6 in which we plot the real frequency \( \omega_r \) (Hz) and growth frequency \( \gamma \) (Hz) for the parameters shown in Table 1 and which pertain to the time period 0225–0233 UT. In Figure 6a we plot the frequencies versus the propagation angle \( \theta \) for the fastest growing mode; this is caused by ion ring 1. The parameters are the same as in Figure 3 except we use \( \omega_{pe}/\Omega_e = 30 \), which, in fact, does not change the results. We find that this mode can generate turbulence in the frequency range \( f_r \approx 10–400 \) Hz. The real frequency at maximum growth is in the range \( f_r \approx 10–40 \) Hz, which is in excellent agreement with the low-frequency peak observed in Figure 5b. The wavelength of this mode is \( \lambda \approx 0.8 \) km. In Figure 6b we plot the frequencies versus the wavenumber \( k\lambda_{de} \) for the electron cyclotron mode generated by ion ring 3. The parameters are the same as in Figure 4 except that \( \omega_{pe}/\Omega_e = 30 \); this causes a shift to smaller wavenumbers for the instability and the first four electron cyclotron harmonics are evident. These harmonics correspond to real frequencies \( f_r \approx 430, 860, 1290, \) and \( 1720 \) Hz. Maximum growth occurs in the range \( f_r \approx 800–1600 \) Hz which, again, is in very good agreement with the wave data shown in Figure 5b.
5. Summary

We have shown that multiple-ring ion distributions observed in the central plasma sheet (CPS) can generate short wavelength ($k<1$), electrostatic turbulence with frequencies $\omega \approx kV_j$, where $V_j$ is the velocity of the $j$th ring.

Using a set of parameters from measurements in the CPS, we find that electrostatic turbulence can be generated with wavenumbers $10^{-2} \approx k\lambda_{de} \approx 1.0$, and with real frequencies in the range $0 < \omega / \omega_{pe} < 10$, and growth rates $\gamma / \omega_{pi} > 0.01$ over a broad range of angles relative to the magnetic field ($5^\circ < \theta < 90^\circ$). This corresponds to wavelengths in the range $1 \, \text{km} < \lambda < 10^2 \, \text{km}$ and frequencies in the range $0 < f < 2 \, \text{kHz}$. The fastest growing modes occur for nearly parallel propagating modes ($5^\circ < \theta < 10^\circ$) with $\omega_r \approx \omega_{pi}$ at wavelengths comparable to the electron Debye length. We also find that lower hybrid and electron cyclotron waves can be excited for nearly perpendicular propagating modes ($\theta = 90^\circ$). These wave frequencies are consistent with observations of electrostatic turbulence in the CPS during the time that ion ring distributions were measured. For example, there were seven ion distribution functions analyzed, including those published by Huang et al. [1989] and Chen et al. [1990]. Of the seven, four manifested multiple-ring structures obeying the $\tilde{A}^{1/4}$ scaling law. The other three also had multiple-ring structures but did not obey this scaling law. Thus it seems likely that multiple-ring ion distribution functions occur frequently in the quiet time CPS. The formation of ion rings obeying the $\tilde{A}^{1/4}$ scaling law has been predicted on the basis of a "chaotic scattering" process [Chen et al., 1990; Burkhart and Chen, 1991]. If this explanation is correct, then the observed turbulence in the quiet time CPS is a manifestation of nonlinear particle dynamics exhibited through the differential memory process [Chen and Palmadesso, 1986].

Ion ring distributions may be a cause of broadband electrostatic noise (BEN) [Scarfe et al., 1974; Gurnett et al., 1976; Grabbe and Eastman, 1984], or at least a component of it, in the central plasma sheet or distant magnetotail [Scarfe et al., 1984; Coroniti et al., 1990]. BEN is commonly observed in the plasma sheet boundary layer at frequencies ranging from 10 Hz to near the electron plasma frequency (~10 kHz) with intensities up to a few millivolts per meter. BEN has been observed in the central plasma sheet but is less intense and does not extend to as high a frequency as in the boundary layer [Grabbe and Eastman, 1984]. It has been suggested
that the spectrum of BEN is generated by a combination of instabilities [Cattell and Mozer, 1986]: the low-frequency portion \( f < 500 \text{ Hz} \) is generated by the lower hybrid drift instability [Huba et al., 1978], while the high-frequency portion is generated by an ion beam instability [Grabbe and Eastman, 1984; Akimoto and Omidi, 1986; Schriver and Ashour-Abdalla, 1987]. Roeder et al. [1991] have reported observations of correlated BEN and electron cyclotron emissions in the plasma sheet from AMPTE Ion Release Module (IRM) wave data. They propose that the BEN is produced by an electron acoustic instability generated by an observed low-energy electron beam and that the electron cyclotron waves are caused by a hot electron loss cone instability. An important aspect of this theory is the presence of a cold electron beam.

Berchem et al. [1991] have recently presented simulation results using a two-dimensional electrostatic simulation code. They apply their model to the plasma sheet boundary layer (PSBL) and include four plasma components in their simulations: an energetic ion beam, a hot electron background, a cold electron background, and a cold ion background. A novel feature of this simulation study is that the energetic ion beam was given a perpendicular ring distribution which gave an effective temperature anisotropy of \( T_{\parallel}/T_{\perp} = 5 \). In addition to exciting the electron acoustic instability and the ion/ion two stream instability, they found that electron cyclotron waves were also excited. However, they argue that these waves were not generated as a plasma instability, but rather were simply enhanced thermal fluctuations caused by the ion ring feature of the energetic ion component.

Further work is clearly needed to assess the importance of the ion ring mode in the Earth's magnetotail. First, a more realistic ion distribution should be considered based upon experimental data of the full distribution function, i.e., \( f_i(v_\perp, v_\parallel) \). It is anticipated that thermal effects, both perpendicular and parallel to the ambient magnetic field [Akimoto et al., 1985a, b], could be significant. More important, one needs to determine whether or not the ion rings are ion shells (or partial shells) in \( v_\perp - v_\parallel \) phase space. Also, electromagnetic effects should be included in the dispersion equation because there is evidence of a magnetic component in the low frequency modes. Thus a more detailed linear theory needs to be developed to assess the significance of the non-Maxwellian ion distribution functions observed in the central plasma sheet. Equally important is a nonlinear analysis to estimate the saturation energy of the instability as well as the wave spectrum. In this regard we note that Lee and Birdsall [1979b] have performed nonlinear simulations of a ring instability but for parameters different than those in the Earth's magnetotail. One issue which needs to be resolved is why the wave turbulence does not destroy the ion rings via nonlinear processes (e.g., quasi-linear smoothing). It is conceivable that the integrity of the ion ring (or perhaps partial shell) distributuons are maintained by an external process as suggested by Chen and Palmadesso [1986]. If this is the case, then a steady state situation may evolve in which there is a balance between the waves trying to dissipate the non-Maxwellian features and the driving process trying to maintain them. Furthermore, a recent study by Holland and Chen [1991] has shown that non-Maxwellian resonant features can be robust and persist in the presence of wave turbulence. These areas of study are under present investigation.

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