Cassini model rheometry
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Abstract. Rheometry serves as a method for the determination of effective length vectors of short antennas by means of electrolytic tank measurements. This paper reports on the application of rheometry to the three linear monopoles mounted for the purposes of the Radio and Plasma Wave Science Experiment on the Cassini spacecraft, which will fly to planet Saturn. The voltage signals induced by incoming waves from the Saturnian radio emissions will be recorded for further evaluation. By direction-finding techniques one will trace back from the collected data to the source regions of the received radio waves and determine the wave polarization. An accurate direction finding is only possible if the effective length vectors of the antennas, which are affected by the spacecraft body, are known to a certain degree of accuracy. It is investigated how rheometry enables the determination of the effective length vectors with the help of a scale model. After a detailed discussion of the fundamentals of rheometry, the application of rheometry to the Cassini scale model is described. The results of the measurements are graphically depicted and discussed with the requirements for direction finding taken into consideration. Finally, an overview of the inflight antenna calibration is given, which will be possible by utilizing the strong Jovian radio emissions during Cassini's Jupiter flyby.

1. Introduction
Cassini will be the fourth spacecraft visiting the planet Saturn. Its mission, unlike that of the former Pioneer 11 and Voyager 1 and 2, will mainly concentrate on Saturn. One major object of the Radio and Plasma Wave Science (RPWS) Experiment on Cassini is the investigation of Saturnian radio emissions in the low-frequency range, roughly from 1 kHz up to 1 MHz. This is accomplished by direction-finding techniques, which are meant to infer the parameters of waves incident upon the spacecraft from the received voltage signals, and further, to trace back to the source locations (the determined wave parameters include not only the direction of propagation but also the wave polarization). For that purpose, there are three linear antenna elements (each 10 m in length) mounted on Cassini, thereby allowing direction finding without spinning the spacecraft [Lecacheux, 1978]. The antennas are electrically short, i.e., short compared with the wavelength of radio waves in the frequency range of interest. The most important quantity in dealing with short antennas is the effective length vector. It represents the directional dependence of the reception properties of an antenna (the receiver input impedance is very high so that the antenna impedance plays no role). It is crucial to the direction-finding analysis that the antennas be short. An accurate direction finding is very difficult in the high-frequency range, as effective length vectors get complex and direction and frequency dependent. Ortega-Molina and Daigne [1984] studied these effects for linear antennas with regard to the Voyager Planetary Radio Astronomy (PRA) experiment, based on King [1956]. Ortega-Molina and Lecacheux [1991, p. 11,451] stated that “PRA instrumental characteristics (channel bandwidth and spacing, level of interferences, sensitivity, polarization response) change drastically near 1–2 MHz.” The variation of effective length vectors in the high-frequency range plays an important role in this context, which cannot easily be taken into account. That is why direction finding using the short antenna approximation is only valid for frequencies up to some megahertz. An accurate direction finding in this frequency range requires the exact knowledge of the antenna’s effective length vectors, as has been confirmed in the course of the Voyager PRA data evaluation [Leblanc and Daigne, 1985].

There are, in principle, three methods conceivable for the determination of effective length vectors: numerical calculations, experiment, and calibration. Numerical calculations would be a formidable task. For example, one could solve the electric field inte-
integral equation for the surface currents on antennas and spacecraft body in the transmission case, where a certain antenna is driven and the others are left open. The results could then be used to determine the corresponding effective length vector. For the experimental approach one builds a model which is in scale with the real spacecraft. In order to investigate the reception properties of an antenna on the spacecraft at a certain frequency, one can use the model as a receiver of waves the wavelength of which, compared with the real receiving situation, is reduced by the same scale factor as the model. Such measurements were performed by Riddle [1976] for the Voyager PRA antennas, which showed that this method is not as straightforward as one could suppose. Employed frequencies lay in the order of 100 MHz and above. Dealing with quasi-static fields would make experiments much easier. Utilizing the fact that the effective length vector of a short antenna does not change down to very low frequencies (actually, to the electrostatic limit; see explanation below), a method applying quasi-static fields of about 1 kHz in an electrolytic tank can indeed be developed for the determination of the effective length vector. The name “rheometry” is borrowed from former authors concerned with this method; the validity of the electrostatic methods is briefly discussed by Hoang [1972] and the experimental performance by Hulin and Epstein [1973], both of which are within the context of “Projet Pyramide.”

The principles underlying the method of rheometry are introduced in section 2. Section 3 reports on the rheometry measurements as applied to a scale model of the Cassini spacecraft. The design of the model, including the RPWS antennas, is described, and the measurement procedure for the determination of the effective length vectors of the antennas is explained in detail. The results are presented in tabular and graphical form, and the given interpretation is supplemented by an error discussion. Finally, section 4 surveys the calibration of the RPWS antennas that is planned during Cassini’s Jupiter flyby by utilizing the strong radio emissions from Jupiter, which are well known from former Voyager missions.

2. Method of Rheometry

2.1. Determination of Effective Length Vectors

The investigation of the reception properties of antennas can be considerably simplified for electrically short antennas, i.e., for those which are much smaller than the wavelength of the received electromagnetic wave, by the concept of a vectorial effective length vector. The antenna may be part of a system of antennas and parasitic conductors. As long as the whole system is very small in extent compared with the wavelength, each antenna included in the system can be treated as a short antenna. It is characteristic of a short antenna that a constant real vector, the effective length vector \( \mathbf{h}_{\text{eff}} \), can be associated with it, so that the voltage \( V \) induced at the open terminals of the receiving antenna is given by

\[
V = \mathbf{E} \cdot \mathbf{h}_{\text{eff}},
\]

with \( \mathbf{E} \) as the electric field strength of the incident wave. The vector is constant (independent of frequency and direction of the incident wave) only in the frequency range where the antenna can be considered to be short. For an isolated linear antenna, \( \mathbf{h}_{\text{eff}} \) equals half the vectorial physical length (parallel to the antenna rods). However, this is not the case when the linear antenna is part of a more complicated system since, in general, \( \mathbf{h}_{\text{eff}} \) is altered in its magnitude and tilted away from the physical direction by the influence of parasitic bodies.

Rheometry is an experimental method devised for the determination of effective length vectors of short antennas using an electrolytic tank. For the time being we discuss the measurements as if they were performed in vacuum, and thus the results would immediately apply to the case of the real deployment. Since (1) is valid down to the low-frequency limit, a homogeneous static or quasi-static electric field can be used to simulate the electromagnetic plane wave incident to the antenna system. This facilitates the setup for the measurement. For reasons that will become clear later on, static fields must be excluded, and we are forced to use an ac quasi-static electric field.

According to (1), the effective length vector \( \mathbf{h}_{\text{eff}} \) of the antenna can be determined by turning the antenna into the position of maximum voltage response \( V_{\text{max}} \). Then \( \mathbf{h}_{\text{eff}} \) is parallel to \( \mathbf{E} \) and the magnitude of \( \mathbf{h}_{\text{eff}} \) directly follows from (1). However, this method can only be used to measure the magnitude \( |\mathbf{h}_{\text{eff}}| \), not to obtain the direction of \( \mathbf{h}_{\text{eff}} \) accurately, as the voltage response \( V \) is very insensitive to small deviations of \( \mathbf{h}_{\text{eff}} \) from the direction of \( \mathbf{E} \). For small angles \( \alpha \) between \( \mathbf{E} \) and \( \mathbf{h}_{\text{eff}} \), \( V \) is given by

\[
V = |\mathbf{E}| |\mathbf{h}_{\text{eff}}|(1 - \alpha^2/2).
\]
This behavior is in great contrast to the situation when $\mathbf{h}_{\text{eff}}$ falls into the plane orthogonal to $\mathbf{E}$, so that $V = 0$. If $(\beta - \beta_0) \sin \gamma$ is a small angle between this plane and $\mathbf{h}_{\text{eff}}$, then the voltage $V$ can be approximated by

$$V = |\mathbf{E}| \, |\mathbf{h}_{\text{eff}}| (\beta - \beta_0) \sin \gamma.$$  

Here we introduced $\beta$ as the angle of revolution of the whole antenna system about a fixed axis perpendicular to $\mathbf{E}$, and $\gamma$ as the angle between the revolution axis and $\mathbf{h}_{\text{eff}}$, thereby $\beta_0$ being the value of the revolution angle $\beta$ for zero voltage response. $V$ changes sign when $\mathbf{h}_{\text{eff}}$ traverses the plane orthogonal to $\mathbf{E}$, as it linearly depends on $\beta$ near $\beta_0$. Therefore $\beta_0$ can be determined accurately by observing the behavior of $V$ around the zero response position. Of course, $\beta_0$ does not yield the direction of $\mathbf{h}_{\text{eff}}$ but establishes a plane in the reference frame of the antenna system to which $\mathbf{h}_{\text{eff}}$ is parallel. The measurement can be repeated for different revolution axes to get at least two noncoplanar planes, the intersection of which uniquely determines the direction of the effective length vector $\mathbf{h}_{\text{eff}}$.

2.2. Measurements in Electrolytic Media

So far we have not specified the medium in which the electric field is to be sustained. In fact, it would not be possible to perform the measurements in air, as the voltage at the antenna terminals would be considerably altered when a voltmeter is connected. The reason is that the impedance of the antenna is very high in the quasi-static frequency range. The capacitances of the various antenna configurations on the Cassini model measure a few picofarads in air, corresponding to a reactance in the order of some 100 M$\Omega$ for a frequency of about 1 kHz that we used in our measurements. It is clear that the input impedance of any measuring voltmeter would substantially perturb the measurement conditions. In the electrostatic limit the voltmeter would even short-circuit the antenna terminals, or the input capacitance of the voltmeter would at least prevail over the capacitance of the antenna. This gross influence can be avoided by immersing the model in an electrolyte (conductivity $\sigma$). An electric field then maintains a proportional current density throughout the electrolytic medium,

$$\mathbf{J} = \sigma \mathbf{E}. \quad (4)$$

As a consequence, the antenna impedance is dominated by the resistance of the electrolyte, which is very small compared with the input impedance of the voltmeter. In other words, the current drawn by the voltmeter is negligible compared with the current flowing through the electrolyte, so the former has only a minor effect on the structure of the potential in the electrolyte. We shall resume this argument later for a more precise and quantitative formulation.

When maintaining an electrostatic field through an electrolytic medium, one has to cope with polarization effects, which result in an addition of surface impedance to the resistance of the electrolyte. The surface impedance consists of a resistance and a capacitance in parallel [cf. Einstein, 1951, and references therein]. These effects make it impossible to use an electrostatic field in the electrolyte. However, in employing alternating electric fields, the influence of polarization can be made negligible as it decreases by increasing frequency. Exceeding 2 kHz does not significantly improve the situation, so suitable operating frequencies are in the range of 1–2 kHz (relevant experimental parameters are given by Einstein [1951]). Using too high a frequency would destroy the quasi-static nature of the electric field. For our measurements we therefore decided in favor of a frequency of about 1.03 kHz, where the odd number is chosen for being apart from multiples of the main frequency of 50 Hz.

The homogeneous electric field meeting the simulation condition is generated by a capacitor consisting of two parallel plates in the electrolyte. The electric field between the plates would be perfectly homogeneous only if the plates were of infinite extent. In practice the electric field is sustained in a tank, the short sides of which are metallic, thereby representing the capacitor plates (see Figure 1). The tank is filled with the electrolytic medium. At the four long sides of the tank the time-harmonic electric field strength in the electrolyte, $E(\mathbf{r}, t) = E(\mathbf{r}) e^{i\omega t}$, and that outside in air, $E^{\text{ext}}(\mathbf{r}, t) = E^{\text{ext}}(\mathbf{r}) e^{i\omega t}$, are coupled by the boundary conditions

$$\mathbf{n} \times \mathbf{E} = \mathbf{n} \times E^{\text{ext}}, \quad (5)$$

$$\mathbf{n} \cdot \mathbf{E} = \frac{\varepsilon_0}{\varepsilon} \mathbf{n} \cdot E^{\text{ext}}, \quad (6)$$

where $\mathbf{n}$ is the respective unit surface normal and $\varepsilon$ denotes the complex dielectric constant of the electrolyte defined by ($\varepsilon$ real permittivity)

$$\varepsilon = \varepsilon - i \frac{\sigma}{\omega}. \quad (7)$$
For our purposes the surrounding air is equivalent to vacuum, with permittivity $\varepsilon_0$ and permeability $\mu_0$. Since $\varepsilon_0 \ll |\varepsilon|$, the boundary conditions assert that $E$ is practically tangential to the surface of the tank at its long sides. This is the reason why the electric field is homogeneous to a very good approximation within the electrolytic tank.

2.3. Equivalence of Field Solutions in Measurement and Real Receiving Situations

As mentioned above, $h_{\text{eff}}$ can be inferred from the dependence of $V$ on the antenna orientation in a homogeneous low-frequency electric field. If the measurements could be performed in vacuum, the results would directly give the desired effective length vectors. However, the measurement must be carried out in an electrolyte. How can $h_{\text{eff}}$ be determined under this condition? In order to be able to proceed as proposed for the vacuum case (discussion following (2) and (3)), the following equivalence must hold: The electric field generated under measurement conditions in the electrolyte has the same form as that induced by the incident electromagnetic wave in the analogous situation in vacuum, i.e., they are proportional to each other. In this context, "analogous" means that the position of the antenna system with regard to $E$ is the same. Measured quantities are the voltage $V$ induced at the antenna and the driving voltage $V_d$ between the tank capacitor plates, giving a driving electric field $E = |E| = V_d/l$, where $l$ denotes the distance of the plates from each other. The equivalence implies that the ratio $V/E$ in the measurement is the same as in the analogous real case. Therefore we derive from $V/E$ the same effective length vector $h_{\text{eff}}$ in either case. This fact is fundamental to rheometry.

A rigorous proof of the equivalence could be accomplished by solution of the respective boundary value problems. An estimation of how the solution alters with the transition from the electrolytic medium to vacuum and from electrostatic conditions to higher frequencies (the real situation) can be found by perturbation theory [cf. Noble, 1962, and references therein]. Here we only intend to elucidate the preconditions of the equivalence, not taking polarization effects into account. For this purpose we first collect some relations needed for a comparative discussion.

The electromagnetic field is generally determined by the Maxwell equations and boundary values on the surface enclosing the region of interest. Since the fields are assumed to be harmonic in time, each component of the electric field strength, $E(r, t) = E(r)e^{i\omega t}$, obeys the Helmholtz wave equation,

$$\Delta E + k^2 E = 0,$$

where $k$ is the wave number and $\omega$ is the angular frequency.
with

\[ k^2 = \omega^2 \varepsilon \mu. \]  

The differential equation (8) is homogeneous, as no space charge can exist within the electrolyte: As a consequence of the Maxwell equations, any charge density \( \rho_0 \) would dissipate according to

\[ \rho = \rho_0 e^{-t/T_r}, \]  

where \( T_r \) denotes the relaxation time of the charges,

\[ T_r = \frac{\varepsilon}{\sigma}. \]  

For our measurements, tap water served as electrolyte, its constitutive parameters being \( \varepsilon = 85 \varepsilon_0, \mu = \mu_0 \), and \( \sigma = 0.036 \text{ S/m} \) (\( \varepsilon \) estimated using tabular values from Lide [1992-1993, pp. 6-10] and \( \sigma \) from the resistance measured between the tank capacitor plates). Substituting these values into (11), we get a relaxation time of the order of

\[ T_r = 2 \times 10^{-8} \text{ s}. \]  

\( T_r \) is much smaller than the period \( 2\pi/\omega \approx 10^{-3} \text{ s} \) of the alternating electric field. An induced charge density vanishes in such a short time interval that it does not influence the measurements at all, which justifies the assumption of a charge-free electrolyte. Similarly, a body situated in the electrolyte discharges exponentially in time. This can be seen by expressing the amount of charge \( Q \) on the body by the current passing through its surface:

\[ Q = \oint \mathbf{D} \cdot \mathbf{d}a = \frac{\varepsilon}{\sigma} \oint \mathbf{J} \cdot \mathbf{d}a = \frac{-e}{\varepsilon} \frac{dQ}{dt}, \]  

where \( \mathbf{d}a \) is the vectorial area element, parallel to the outward normal. \( Q \) is thus governed by the differential equation

\[ \frac{dQ}{dt} + \frac{\varepsilon}{\sigma} Q = 0, \]  

the solution of which can be written

\[ Q(t) = Q_0 e^{-t/T_r}, \]  

where \( Q_0 \) denotes the initial value of \( Q \) at \( t = 0 \). The body is losing its charge within the same characteristic time \( T_r \) as any initial charge density within the electrolyte would take to vanish. This holds for any body encompassed by the electrolytic medium, the integral in (13) being taken over a closed surface around the respective body. However, as for the antennas on the Cassini model, there are two conductors at a time (out of three antenna elements and the main body), either of which is connected by one thin wire to the voltmeter that measures the voltage between these conductors. The derivation in (13) is not possible for these conductors, as the current through the leads to the voltmeter must be taken into account. Very thin insulated wires are provided so that the contribution of their surface does not significantly perturb the field around the model. In other words, we can neglect the influence of an open wire (voltmeter disconnected) on the electric field. The effect of a wire between conductor and amplifier is then limited to an alteration of the boundary condition \( Q = 0 \) for the conductor. In order to calculate the sources \( Q \) on such a conductor, we express \( \oint \mathbf{D} \cdot \mathbf{d}a \) by two currents: On the one hand, charges flowing from the electrolyte to the conductor give a current \(-\int_{S_e} \mathbf{J} \cdot \mathbf{d}a \) (\( \mathbf{d}a \) outward directed), where \( S_e \) is that part of the conductor’s surface that borders on the electrolyte. On the other hand, we note the current \( I \) through the wire, which can be written \(-\int_{S_w} \mathbf{J} \cdot \mathbf{d}a \), where \( S_w \) is the wire’s cross section at the joint to the conductor. The union of \( S_e \) and \( S_w \) constitutes a closed surface around the body of the conductor, so we can do this rearrangement:

\[ \oint \mathbf{D} \cdot \mathbf{d}a = \frac{\varepsilon}{\sigma} \oint_{S_e} \mathbf{J} \cdot \mathbf{d}a + \frac{\varepsilon_c}{\sigma_c} \oint_{S_w} \mathbf{J} \cdot \mathbf{d}a \]

\[ \oint \mathbf{D} \cdot \mathbf{d}a = \frac{\varepsilon}{\sigma} \oint \mathbf{J} \cdot \mathbf{d}a + \left( \frac{\varepsilon_c}{\sigma_c} - \frac{\varepsilon}{\sigma} \right) \oint_{S_e} \mathbf{J} \cdot \mathbf{d}a. \]  

Here \( \varepsilon_c \) and \( \sigma_c \) denote the conductor’s permittivity and conductivity, respectively. The model and the wires are usually made of excellent conducting materials (see next section for model description); \( \sigma_c > 10^8 \sigma \). Therefore \( \varepsilon_c/\sigma_c \) can be omitted inside the parentheses as it has no material contribution to \( \varepsilon/\sigma \). Equation (16) expresses the charge \( Q = \oint \mathbf{D} \cdot \mathbf{d}a \) on the conductor by its time-derivative \( \dot{Q} = -\oint \mathbf{J} \cdot \mathbf{d}a \) and the current \( I = -\oint_{S_e} \mathbf{J} \cdot \mathbf{d}a \) through the lead toward the conductor, leaving the differential equation

\[ \frac{dQ}{dt} + \frac{\varepsilon}{\sigma} Q = I. \]  

Comparison with (14) shows that the current through the lead introduces an inhomogeneity to the differ-
ential equation for \( Q \). Under measurement conditions, an ac voltage is applied to the driving plates. Two conductors at a time (antenna-antenna or body-antenna) are connected by leads to the voltmeter recording the voltage \( V \) between these conductors. The charge on either conductor satisfies (17), which becomes

\[
(i \omega + \sigma / \varepsilon) Q = \frac{V}{Z}
\]

in the time-harmonic case. The current \( I \) over the voltmeter has been expressed by the measured voltage \( V \) and the input impedance \( Z \) of the voltmeter. In order to settle whether \( Z \) affects the measurement or not (discussed in the next paragraph), we compare the measurement situation with the following one: The model is placed in the field-free electrolyte. Then the voltage \( V \) which has been measured before is applied to the conductors previously connected to the voltmeter. The current through the leads is \( V/Z' \), where \( Z' \) is the impedance of the model-electrolyte system for the respective antenna configuration driven by \( V \). \( Z' \) consists of the resistance \( R \) from the electrolytic medium and the capacitance \( C \) of the antenna configuration, in parallel. Thus the charges \( Q' \) on each of the two driven conductors of the model obey (18), with \( Q \) and \( Z \) substituted by \( Q' = CV \) and \( Z' = (1/R + i\omega C)^{-1} \), respectively, giving

\[
\frac{\sigma}{\varepsilon} Q' = \frac{V}{R}
\]

Reordering further yields

\[
RC = \frac{\varepsilon}{\sigma}
\]

\[
Z' = \frac{R}{1 + i\omega \varepsilon / \sigma}.
\]

Since \( \omega \varepsilon / \sigma \) is about \( 10^{-4} \) for our measurements, \( Z' \) is essentially real and given by \( R \). The values found by experiment vary from antenna to antenna, but all have the same order of magnitude, \( R \approx 100 \ \Omega \). The corresponding capacitances follow from (20). By substituting \( \varepsilon_0 \) for \( \varepsilon \), we get the vacuum-capacitances, which are of the order of a few picofarads.

We now return to the discussion of the equivalence as stated above. First consider the boundary conditions for the model's surface. Since the conductivity of the model is virtually infinite compared with that of the electrolyte, the electric field can be assumed perpendicular to the surface. In addition, the charges \( Q \) on any separate part (three antennas and body) of the model vanish. We have shown that this is true for conductors completely enclosed by the electrolyte. For a conductor connected to the voltmeter, we arrived at (18) by allowing for a current through the leads. When the antenna is driven in the otherwise field-free electrolyte with just that voltage that is induced under measurement conditions, a charge \( Q' \) obeying (19) is produced on either part of the antenna. If the charges \( Q \) induced under measurement conditions are negligible compared with \( Q' \) (\( Q \ll Q' \)), we can suppose that \( Q \) does not significantly perturb the distribution of the electric field, i.e., the condition \( Q = 0 \) amounts to a correct solution for \( E \) in the measurement situation. In fact, we find a ratio

\[
\frac{Q}{Q'} \ll \frac{Z'}{Z} \approx \frac{R}{1}
\]

with magnitude \( |Q/Q'| \approx 10^{-5} \), where a realistic value of the magnitude \( |Z| \) of the input impedance of the voltmeter of the order of 10 M\( \Omega \) is assumed. Hence the voltmeter does not interfere with the ideal measurement conditions. This is only valid on the condition that the conductivity of the electrolyte is large enough to hold \( R \ll Z \) (it does not apply, of course, when the electric field is maintained in air). Eventually, we find the same boundary values in the electrolyte and in vacuum: The total charge on any conductor vanishes \( (Q = 0) \), with \( E \) being perpendicular to its surface.

It is left to compare the governing differential equations. As in the electrolyte, \( E \) obeys a Helmholtz equation (8) in vacuum but with \( k \) substituted by \( k_0 = 2\pi/\lambda \). The vacuum wavelength \( \lambda \) of the received electromagnetic wave is much greater than the dimension \( d \) of the antenna system since short antennas are assumed. As long as the wavelength satisfies this condition, the antenna effective length vector \( \mathbf{h}_{\text{eff}} \) is representative of the reception properties of this antenna, as explained above, independent of frequency down to the electrostatic limit. Since \( E \) is Laplacian in the electrostatic limit, this argument shows that assuming the antenna to be short amounts to neglecting \( k^2 E \) in the Helmholtz equation. Transferring the relation \( \lambda \gg d \), or \( k_0 \ll 2\pi/d \), to the case of an electrolytic medium, we can suppose

\[
|k| \ll \frac{2\pi}{d}
\]
as a precondition of $E$ to be Laplacian. For tap water and $\omega \approx 2\pi \times 10^3 \text{ s}^{-1}$, we obtain from (9) and (7) $k^2 = (4 \times 10^{-8} - 3 \times 10^{-4} i) \text{ m}^{-2}$. Models of antenna systems, which are to be investigated by rheometry, are usually built to such a scale that their dimension is typically about half a meter or less. Thus we have $|k| \approx 10^{-3} (2\pi/d)$, confirming that (23) is well fulfilled. Therefore we can finally summarize that all found preconditions of the initially stated equivalence definitely hold, not only with respect to boundary values but also with respect to the governing differential equation.

3. Rheometry for the Cassini Model

3.1. Model Description

From the Cassini spacecraft configuration layout released by the Jet Propulsion Laboratory (JPL), a model configuration was derived on the scale 1:30. Drawings of the model in elevation can be found in Figure 2. The figure also shows how the spacecraft-fixed coordinate system is defined: the $x$ axis opposite to the Huygens probe, the $y$ axis in the direction of the magnetometer boom, and the $z$ axis opposite to the high-gain antenna. In this reference frame, spherical
Table 1. Effective Length Vectors $h_{\text{eff}}$ of the Radio and Plasma Wave Science Experiment Antennas Onboard the Cassini Spacecraft as Found by Rheometry Measurements

<table>
<thead>
<tr>
<th>Antenna</th>
<th>Physical $h$</th>
<th>$\theta$</th>
<th>$\varphi$</th>
<th>$h_{\text{eff}}$ - HP on $h$</th>
<th>$\theta$</th>
<th>$\varphi$</th>
<th>$h_{\text{eff}}$ - HP off $h$</th>
<th>$\theta$</th>
<th>$\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+z</td>
<td>5.0</td>
<td>37.0</td>
<td>90.0</td>
<td>4.4</td>
<td>31.4</td>
<td>91.2</td>
<td>4.4</td>
<td>30.8</td>
<td>92.9</td>
</tr>
<tr>
<td>+x</td>
<td>5.0</td>
<td>107.5</td>
<td>24.8</td>
<td>4.0</td>
<td>107.9</td>
<td>16.5</td>
<td>4.0</td>
<td>107.6</td>
<td>16.3</td>
</tr>
<tr>
<td>−x</td>
<td>5.0</td>
<td>107.5</td>
<td>155.2</td>
<td>4.0</td>
<td>107.3</td>
<td>162.7</td>
<td>4.0</td>
<td>106.4</td>
<td>163.5</td>
</tr>
<tr>
<td>Dipole</td>
<td>8.7</td>
<td>90.0</td>
<td>0.0</td>
<td>7.3</td>
<td>90.5</td>
<td>359.8</td>
<td>7.4</td>
<td>90.6</td>
<td>359.9</td>
</tr>
<tr>
<td>Common</td>
<td>2.5</td>
<td>127.0</td>
<td>90.0</td>
<td>1.6</td>
<td>137.9</td>
<td>88.6</td>
<td>1.6</td>
<td>137.9</td>
<td>91.0</td>
</tr>
</tbody>
</table>

The given values represent spherical coordinates in the spacecraft-fixed reference frame (see Figure 2), with colatitude $\theta$ and azimuth $\varphi$ in degrees and magnitude $h = |h_{\text{eff}}|$ in meters. The column headed "physical" contains the corresponding effective length vectors for the imaginary case of an infinitely small spacecraft body, whereby the angular values for +z, +x and −x represent the direction of the physical antenna elements. Abbreviations are HP, Huygens probe, and RTG, radioisotope thermoelectric generator.

The model was developed under a number of aspects. The shape of the model should be as close as possible to reality without making it too fragile for handling during the rheometry measurements. The model is suspendable in three positions, one axis of the spacecraft-fixed reference frame being vertical. Three boreholes are made for this purpose. Focus of attention was laid on specific spacecraft structures which might influence the reception properties of the antennas. These are the removable Huygens probe (HP), the cosmic dust analyzer (CDA), and the shades of the radioisotope thermoelectric generators (RTG). The HP will be jettisoned toward Titan before Cassini's Saturn orbit insertion. This is the reason why it is crucial to study the influence of the HP removal on the effective length vectors of the antennas. The CDA can be swiveled round a fixed axis and comes rather close to the −x antenna at certain positions. Similarly, one of the RTG shades is close to the +z antenna.

The model is made of brass and is gold-plated (as are the antennas). The gold coat ensures excellent conducting properties and minimum reaction with the water during the measurements. The antennas are insulated from each other and from the spacecraft coordinates with colatitude $\theta$ and azimuth $\varphi$ are used to represent the direction of the antennas mounted on the spacecraft. There are three linear monopoles, as shown in Figure 2, denoted by +z, +x, and −x. Each antenna rod is 10 m in length (33 cm for the model), which is about twice the extent of the spacecraft body. The +x and −x antennas exhibit an opening angle of 120°, both perpendicular to the +z antenna, which is tilted 37° with regard to the z axis in the yz plane. It is important not to confuse the antennas +x and +z with the axes x and z, respectively. The direction of the physical antennas in spherical coordinates can be found in Table 1. The word "physical" emphasizes that antenna rods have to be distinguished from their corresponding effective length vectors. In general, the column headed "physical" gives the lengths and directions of $h_{\text{eff}}$ of the antennas as assumed in the case of an infinitely small perturbing spacecraft body. For the monopoles these ideal effective length vectors coincide with half the vectorial length of the antenna rods. The monopoles are driven/measured versus the spacecraft body. The "dipole" is defined as +x versus −x, and the "common mode" as +x and −x short-circuited, versus the spacecraft body. Therefore each of these five antenna configurations leaves certain conductors open; for example, the −x and +z rods remain open for the +x monopole and the +z rod and the spacecraft body for the dipole configuration.
body, as can be seen from the Cassini model photographs which are exhibited in Plate 1.

3.2. Experimental Performance

Figure 1 shows the setup for the rheometry measurements. The electric field is sustained in an electrolytic tank of size $2 \times 1 \times 1$ m, which is filled with tap water up to more than 95 cm above ground. The conductivity $\sigma$ of the water is measured as $\sigma = 0.036$ S/m. The walls of the tank are made of marble, ensuring the tank to be robust enough to hold 2 t of water. Using steel would not be possible because the walls must provide nonconducting borders at 5 sides (including the bottom) of the water. Steel plates are attached at the inner surfaces of the short sides of the tank, building a capacitor together with the water. This capacitor is driven by an alternating voltage of about 0.4 V rms and 1.03 kHz, maintaining a homogeneous alternating electric field throughout the water.

The model is lowered into the middle of the tank and supported by a device made of nonconducting synthetic material to avoid distortion of the electric field in the tank. The support consists of three major parts: the “sandwich,” the model carrying tube, and the angular scale. The sandwich lying across the tank as shown in Figure 1 is composed of two transparent plates, which are separated by two rails, preventing the plates from bending under the model’s load. The model is suspended from the sandwich by a vertical tube which can be fixed at any height. The angular scale is then joined with the tube to provide a defined turning of the model. The reading of the scale directly gives the rotation angle of the model around the vertical axis. The model can be suspended in three orthogonal positions, one axis of the model-fixed reference frame being vertical: $+x$ axis upward, $+y$ axis downward, and $+z$ axis upward.

The procedure for determining the effective length vectors $b_{\text{eff}}$ of the various antenna configurations is as follows: Before immersing the model into the water-filled tank, the three antenna elements have to be carefully aligned with their respective nominal directions in the model’s reference frame. By means of a special calibration device this orientation is possible within an error of about 0.5°. The model is brought into measurement position using one of the three possible suspensions, for example, with the $+y$ axis pointing downward as shown in Figure 1. Very thin wires (diameter 0.28 mm) are used to connect the antenna terminals to the lock-in amplifier, which serves as a highly sensitive voltmeter, thereby rejecting noise signals off the generator frequency (in particular, making the experiment insensitive to con-
tingent contact potentials). Then, the following measurements are performed for each antenna, starting with the calibration of the angular scale. The calibration is realized by setting the angular scale so as to read 0° in case the antenna comes to lie parallel with the capacitor plates. While the model is turned once around the rotation axis, the voltage response reading from the lock-in amplifier crosses two zeros according to (1). The angles \( \eta_1 \) and \( \eta_2 \) of zero response occur when \( \mathbf{h}_{\text{eff}} \) stands orthogonal to \( \mathbf{E} \), i.e., parallel with the capacitor plates. Thus the measured values should differ by 180°, and one representative value \( \eta \) is derived by averaging

\[
\eta = \frac{\eta_1 + \eta_2 - 180°}{2}. \tag{24}
\]

The maximum rms voltage responses \( V_{\eta_1} \) and \( V_{\eta_2} \) are expected for the angles \( \eta + 90° \) and \( \eta - 90° \). The values \( V_{\eta_1} \) and \( V_{\eta_2} \) measured at these angles result in a representative mean value

\[
V_{\eta} = \frac{V_{\eta_1} + V_{\eta_2}}{2}. \tag{25}
\]

This procedure is performed with the three monopoles \(+x, -x, +z\), the dipole \((+x \text{ versus } -x)\), and the common mode \((+x \text{ and } -x \text{ short-circuited, versus spacecraft})\). The measurements described so far are repeated in the same way for other suspensions of the model, \(+x\) upward and \(+z\) upward, yielding zero response angles \( \xi \) and \( \zeta \) and maximum response voltages \( V_{\xi} \) and \( V_{\zeta} \), respectively.

Putting the measurements together, there are six quantities \((\xi, \eta, \zeta, V_{\xi}, V_{\eta}, \text{ and } V_{\zeta})\) per antenna to calculate its effective length vector \( \mathbf{h}_{\text{eff}} \), which is determined by three values only (\( \theta \) and \( \phi \) represent the direction of \( \mathbf{h}_{\text{eff}} \) and \( h = |\mathbf{h}_{\text{eff}}| \) its length). In consequence, this overdetermination enables us to examine the consistency of the measurements.

### 3.3. Results and Discussion

A number of rheometry test series have been performed with gradually improved model design and ever-increasing accuracy of results (Observatoire de Paris-Meudon, 1993; Observatory Graz-Lustbuehel, 1994). The final results are given in Table 1. The spherical coordinates \((h, \theta, \phi)\) of the effective length vectors are calculated from the six measured quantities \( \xi, \eta, \zeta, V_{\xi}, V_{\eta}, \text{ and } V_{\zeta} \) by means of a least squares fit. The measurements are performed with four constellations, HP and RTG shades being mounted on the spacecraft or taken off, respectively.

The general tendency of the \( \mathbf{h}_{\text{eff}} \) directions of the monopoles can be seen from Figure 2. They are tilted away from the respective physical elements toward the spacecraft body. More precisely, each monopole possesses an \( \mathbf{h}_{\text{eff}} \) closely lying in the plane spanned by its physical element and the \( y \) axis, with \( \mathbf{h}_{\text{eff}} \) tilted out of the physical element direction away from the \( y \) axis. The angular offsets measure 5.6°, 7.9°, and 7.2° for the \(+z, +x, \text{ and } -x\) antennas, respectively. These values are surprisingly small compared with the corresponding offsets ascertained for the two antennas of the Planetary Radio Astronomy Experiment (PRA) on Voyager. Lecacheux and Ortega-Molina [1987] found by calibration using Uranian kilometric radiation (UKR) that the electrical plane defined by the effective length vectors is tilted by 23.3° from the physical plane of the antennas and that the effective length vectors enclose an angle of 100°, the physical antenna elements being orthogonal. Hence the offsets for the individual PRA antennas amount to 17.8°. We have no analytic expression which could explain the discrepancy between our results for Cassini and those evidenced for Voyager. Instead, we list the following facts which make plausible why such a large difference in the offsets can occur. Cassini and Voyager essentially differ first in the shape of the main body (e.g., Cassini is elongate in contrast to Voyager) and second in the mounting position and the orientation of the antennas with regard to the spacecraft body (e.g., the \(+x\) and \(-x\) Cassini antennas enclose an angle of 120°, whereas the PRA antennas are orthogonal). In comparing the physical plane of the \(+x\) and \(-x\) Cassini antennas with that of the Voyager PRA antennas, we note an angular distance from the respective symmetry axes of 40° on Voyager and 53° on Cassini. Then it is conceivable that the offset of the Cassini electrical plane is the smaller one when we extrapolate to the case where the physical plane stands orthogonal to the spacecraft symmetry axis. In this case the offset should nearly vanish for reasons of symmetry, provided that the antennas are attached in the middle of the longitudinal axis, which fairly applies to the Cassini antennas.

Redundant measurements typically yield \( \mathbf{h}_{\text{eff}} \) directions with a mean deviation of about 1° from each other. Turns of the CDA have no observable effect, so they influence the effective length vectors on a scale much smaller than the measurement precision of 1°. The HP removal significantly affects only the \(-x\) and
+z monopoles. With the removal, the effective length vectors of both are bent toward the HP attachment by about 1°. The alteration of \( \mathbf{h}_{\text{eff}} \) by taking off the RTG shades is negligible for all antennas except the +z monopole. Even the \( \mathbf{h}_{\text{eff}} \) direction of the +z monopole is only changed by an angle of just the order of the measurement precision. Comparison of the results obtained from measurements with and without RTG shades enables an estimate on how great an error in the predicted \( \mathbf{h}_{\text{eff}} \) can be caused by possible insufficiencies in the spacecraft modeling. Since details not exactly modeled are in the dimension of the RTG shades at most, we can suppose that the error due to the modeling is smaller than 1°. An additional source of inaccuracy not accounted for is the deviation from the ideal measurement conditions (i.e., no polarization effects, model placed in an infinite homogeneous electric field). Some work on these effects has been done, showing so far that they cause no inaccuracies larger than the measurement precision. Hence we eventually estimate the results of rheometry to be accurate to within 2°.

4. In-Flight Calibration of the RPWS Antenna System and Direction Finding

Although rheometry results are attractive, there exist possibilities to check and to complement the results by performing in-flight antenna calibration using the strong radio signals from the planet Jupiter. On its way to Saturn the Cassini spacecraft will encounter Jupiter during the year 2000, thereby collecting data from the Jovian radio emissions, the kilometric radiation of which being best suited to the calibration needs because of its large wavelength. The properties of the Jovian radio emissions, in particular, the source positions and beaming properties, are well known from former Voyager [Boischot, 1988, and references therein] and Ulysses missions [Reiner et al., 1993a, b; Ladreiter et al., 1994]. We thus are able to select time periods for roll maneuvers of the Cassini spacecraft (around its z axis) during which the antenna response of a given antenna is modulated in such a way that it crosses two zeros per complete turn. A zero response occurs when the effective length vector \( \mathbf{h}_{\text{eff}} \) of the antenna points toward the Jovian radio source, since \( \mathbf{h}_{\text{eff}} \) then stands orthogonal to the electric field strength \( \mathbf{E} \) of the incoming electromagnetic wave. We already learned from (3) that such geometries are required since they allow the \( \mathbf{h}_{\text{eff}} \) direction to be accurately determined.

Figure 3 shows the colatitude \( \theta_J \) of Jovian position as seen from Cassini around Jupiter closest approach (CA). The periods where calibration of the respective antennas is appropriate are emphasized by dashed vertical lines. The +x and −x antenna calibrations have to be performed close to the Cassini-Jupiter encounter, whereas the +z antenna has to be calibrated some 75 days before and 40 days after the encounter.
good initial values, which can be provided by the rheometry results. Calibrations using Jovian radio sources will be performed when the HP is still onboard the spacecraft, whereas data aquisition from Saturnian radio waves will begin after the HP will have been jettisoned toward Titan. Therefore it is particularly essential to know the alteration of effective length vectors with the HP removal.

5. Conclusion

Rheometry enables the determination of the effective length vectors of short antennas by investigations using an electrolytic tank. It is possible to simulate the wave incident to the short antenna by a low-frequency (quasi-static) electric field, as the antenna effective length vector concept is valid, independent of frequency, down to the electrostatic limit. Since the antenna impedance is too high in air to enable a measurement of the induced voltage, an electrolytic environment is provided so as to prevent, unlike in air, the connected voltmeter from affecting the experimental conditions.

Rheometry measurements have been performed for the Radio and Plasma Wave Science Experiment antennas on the Cassini spacecraft by means of a scale model. The results show that the effective length vectors of the three monopoles do not coincide with their respective physical elements but are tilted toward the spacecraft body, enclosing an angle with the magnetometer boom (y axis) larger than the respective physical elements do (offset of about 6° for +z and about 7°-8° for +x and −x). The removal of the Huygens probe effects an alteration of the \( \mathbf{h}_{\text{eff}} \) direction of the −x and +z monopoles by about 1°. Taking off the shades of the radioisotope thermolectric generators causes only directional changes within the order of the measurement precision. No influence is observed in turning the cosmic dust analyzer.

An inflight calibration of the antennas can be performed during the Jupiter flyby by utilizing the strong radio emissions from the planet. This will enable a confirmation of the rheometry results. We can expect the results to be in good agreement, but we must not forget that the Cassini model for rheometry has been configured according to a preliminary version of the spacecraft design. So deviations larger than the directional accuracy of about 2° of the effective length vectors found by rheometry could be possible. Inflight calibration will be able to render an adaptation of the effective length vectors, thus this procedure is highly recommended.

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