Quantitative test of the cavity resonance explanation of plasmaspheric Pi2 frequencies

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A Pi2 wave event was observed by the CRRES satellite on day 48 (17 February) of 1991, 1220–1250 UT [Takahashi et al., 2001]. Using the electron density profile measured by CRRES on its outward pass (during which the wave event was observed), and assuming that the ions are all H+, we calculated the theoretical plasmaspheric cavity resonance frequencies using a two-dimensional MHD simulation in dipole geometry (azimuthal symmetry). Our theoretical frequencies are in good agreement with the observed frequencies, and this supports the cavity resonance mechanism as an explanation for the observations. The observed and theoretical harmonic frequency ratios \( f_2/f_1 \) and \( f_3/f_1 \) were in rough agreement with earlier studies. Comparison of the outbound and inbound legs of the CRRES orbit on 17 February 1991 shows that the density on the outbound leg at magnetic local time (MLT) = 20–23.4 was different than that on the inbound leg at MLT = 23.4–4.5. On the outbound leg an enhanced density region beyond the region of the nominal plasmapause (which may have been part of a plasmaspheric tail [Sandel et al., 2001]) played an important role in determining the cavity resonance frequencies. Observations by the Imager for Magnetopause-to-Aurora Global Exploration (IMAGE) spacecraft have revealed that there is often a great deal of azimuthal structure in the plasmasphere (including such tails). Such structure may play an important role in determining observed Pi2 frequencies.

INDEX TERMS: 2752 Magnetospheric Physics: MHD waves and instabilities; 2768 Magnetospheric Physics: Plasmasphere; 2740 Magnetospheric Physics: Magnetospheric configuration and dynamics; 7827 Space Plasma Physics: Kinetic and MHD theory; KEYWORDS: Pi2, cavity mode, fast mode resonance, plasmasphere, MHD, CRRES

1. Introduction

The Pi2 designation refers to impulsive pulsations with periods ranging from 40 to 150 s [Jacobs, 1970], and the phenomenology of Pi2s has been well reviewed by Olson [1999]. As discussed by Olson, the Pi2 signal encompasses a class of pulsations related to two fundamental processes, a sudden generation of the field-aligned currents in association with the disruption of cross-tail currents in the plasma sheet and the impulsive response of the inner magnetosphere to the compressional waves that are generated at substorm onset.

The compressive Pi2 pulsations observed at low and middle latitudes (mapping to the plasmasphere) are often found to have several discrete frequencies. These have been attributed to surface waves at the plasmapause [Sutcliffe, 1975] and to temporal variations of magnetotail flow bursts [Kepko and Kivelson, 1999]. However, the most common explanation for these frequencies is that they are the resonant frequencies of the fast/magnetosonic wave trapped within the plasmasphere. These resonances were originally called cavity modes [Allan et al., 1986; Yeoman and Orr, 1989; Zhu and Kivelson, 1989; Lee, 1996], though it should be understood that they are really leaky cavity modes due to energy loss across the plasma trough and magnetopause [Fujita and Glassmeier, 1995; Lee, 1998; Lee and Kim, 1999]. For this reason, Lee [1998] and Lee and Kim [1999] refer to cavity resonances as “virtual resonances.” Besides explaining the apparent harmonic structure of the observed frequencies, the cavity resonance model explains why dayside compressive waves with similar properties are often observed in association with nighttime Pi2 [Sutcliffe and Yumoto, 1991].

Despite the encouraging features of the plasmaspheric cavity model, it has never been adequately shown that the Pi2 frequencies observed are equal to the resonant frequencies of the plasmasphere. The reason for this is that the plasmaspheric density must be known to accurately calculate the resonant frequencies, and the plasmaspheric density is not usually readily available. Simulations usually assume

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a model density profile that may not accurately represent the density at a particular time [e.g., Cheng et al., 1998, 2000; Lee and Lysak, 1999; Fujita et al., 2000, 2001]. One encouraging study examined a plasmaspheric oscillation following a sudden storm commencement and predicted the frequency of the oscillation using the density profile measured by the Polar spacecraft [Goldstein et al., 1999]. However, the single oscillation observed was not enough to conclusively show that a cavity-like oscillation was involved. Several studies have predicted the ratios of harmonic frequencies, and these will be discussed later.

[5] Here, we examine the observation of a plasmaspheric Pi2 wave by the CRRES satellite [Takahashi et al., 2001]. Using the density profile measured by CRRES, we run an MHD simulation to calculate the resonant frequencies of the plasmaspheric cavity. We then compare the simulation frequencies with the observed frequencies. In addition, we present plots showing the eigenmode structure of the first three harmonics. The CRRES data are discussed in section 2; the simulation model and results are described in section 3; some discussion is presented in section 4; and we sum up with conclusions in section 5.

2. CRRES Data

[6] The data for this study were chosen from the Pi2 study of Takahashi et al. [2001]. A Pi2 wave was observed by the CRRES satellite on day 48 (17 February) of 1991, 1220–1250 UT (event D of the study of Takahashi et al.). The CRRES data were analyzed in terms of mean-field-aligned coordinates, where $\mathbf{e}_z$ is the direction of the mean magnetic field, $\mathbf{e}_r$ is the direction $\mathbf{e}_z \times \mathbf{r}$, where $\mathbf{r}$ is the radial position vector of the spacecraft, and $\mathbf{e}_\theta = \mathbf{e}_z \times \mathbf{e}_r$. Note that $\mathbf{e}_\theta$ approximately represents the azimuthal direction. For more details, see the work of Takahashi et al. [2001].

[7] Figure 1 shows the time-dependent traces of the azimuthal components of electric field $E_y$ (solid curve) and the compressional component of magnetic field $B_z$ (dashed curve) for this event. These two components are $\sim 90^\circ$ out of phase, and this indicates that there is standing wave structure in the radial direction (the Poynting vector associated with these two components is in the radial direction). Other pairs of components of electric and magnetic field did not indicate standing wave structure. Therefore it appears that the wave energy may be trapped only in the radial direction and unconfined along the magnetic field and azimuthal directions.

[8] CRRES had an elliptical orbit with an apogee of 6.3 $R_E$. On day 48, 1991, the apogee of CRRES occurred at $\sim 1315$ UT at MLT $= 23.4$. The wave event shown in Figure 1 was observed when CRRES was near apogee ($L = 6.3$, MLT $= 23.1$, MLAT $= -5^\circ$) on the outbound portion of the orbit.

[9] Figure 2a shows the electron density $n_e$ at the position of CRRES inferred from measurements of the upper hybrid frequency by the CRRES Plasma Wave Experiment [Anderson et al., 1992]. The thin solid curve shows the value of $n_e$ on the outbound leg of the orbit from $L = 3–6.6$. (Here $L$ is the maximum radial distance on the field line based on the T89c magnetic field model [Zyganenko, 1989].) On this outbound portion of the orbit, MLT $= 20–23.4$. The thin solid curve is partially obscured by the thick solid curve, which is our model mass density ($= n_i$, assuming that all ions are $H^+$). The $L$ value at the time that CRRES observed the wave event is indicated by the position of the asterisk in Figure 2b. Comparing Figures 2a and 2b, it is clear that CRRES was just within an extended region of the plasmasphere at the time of the wave event.

[10] The dotted curve in Figure 2a shows the value of $n_e$ on the inbound leg of the orbit (MLT $= 23.4–4.5$). Note that the density on the inbound leg is remarkably similar to that on the outbound leg, except that the outer region of enhanced density ($L = 4.2–6.2$ on the outbound leg) does not extend to as large values of $L$. This outer region of density with increasing radial width toward dusk local time may be a plasma tail (D. Gallagher, private communication, 2000), as has been recently observed by the Imager for Magnetopause-to-Aurora Global Exploration (IMAGE) spacecraft [Sandel et al., 2001]. Our model mass density $\rho$ is based largely on $n_i$ from the outbound leg, but the flatness of the density for $L < 3$ and $L > 6.8$ was based on $n_e$ from the inbound leg.

[11] Figure 2b shows the normalized Alfvén speed $V_A$ versus $L$ based on the model $\rho$ and a dipole magnetic field (normalized so that $V_A = 1$ at the position of CRRES). It is clear that the plasmasphere, including the extended region to $L = 6.8$, forms an Alfvén speed potential well, as discussed by Lee and Kim [1999]. Figure 2c shows the normalized crossing time coordinate for propagation time across $L$ shell, $\tau_A = \int dL/V_A$, versus $L$.

[12] Figure 3a shows the power spectra of CRRES $E_y$ (thick curve) and $B_z$ (thin curve) fluctuations versus frequency $f$ in mHz. (See the work of Takahashi et al. [2001] for more details on the data analysis.) Three peaks are evident (the third is evident only in the $B_z$ power) near the vertical dashed lines marked mode 1, 2, and 3 above Figure 3a. These peaks are clearer in the coherence between $E_y$ and $B_z$ shown in Figure 3b.

3. Simulation Model and Results

[13] In order to estimate the theoretical cavity resonance frequencies, we performed a two-dimensional MHD sim-
ulation in dipole geometry. There were 31 grid points in the field line direction and 41 grid points in the \( L \) direction. At \( L = 6.2 \), corresponding to the position of CRRES, the parallel grid points were distributed evenly with respect to the crossing time coordinate along the field line \( \tau_A = \int dl/V_A \), where \( dl \) is the differential length along the field line. In the \( L \) direction the grid points are distributed evenly with respect to \( L \) shell. There was no dependence in the azimuthal direction (azimuthal mode number \( m = 0 \)). Perfect conductor boundary conditions were applied at the inner \( L \) shell and at the ionospheric boundaries (endpoints of parallel grid). The inner and outer \( L \) shell boundaries were at \( L = 2.5 \) and 7.3, respectively. At \( L = 6.2 \), corresponding to the position of CRRES, the ionospheric boundary is at a geocentric radius of 2 \( R_E \). In Figures 4b–4g the simulation boundaries are indicated by the thick solid curve surrounding the contours. The \( L \) shell boundaries are also indicated by the

**Figure 2.** (a) Electron density \( n_e \) observed by CRRES outbound (thin solid curve) and inbound (thin dashed curve) and model mass density \( \rho \) (thick solid curve); (b) model Alfvén speed \( V_A \) profile; and (c) Alfvén equatorial crossing time \( \tau_A \) versus \( L \). The vertical dashed lines are plotted at the simulation boundary, and the asterisk in Figure 2b is drawn at an \( L \) value corresponding to the position of CRRES at the time of the Pi2.

**Figure 3.** (a) Power spectra of CRRES \( E_y \) in \((\text{mV/m})^2/\text{Hz}\) (thick curve) and \( B_z \) in \(\text{nT}^2/\text{Hz}\) (thin curve) fluctuations, (b) coherence between CRRES \( E_y \) and \( B_z \), (c) power spectra of simulation \( E_y \), and (d) power spectra of simulation kinetic energy versus mode frequency \( f \). The three vertical dashed lines are plotted at the kinetic energy peaks in Figure 3d, and the mode labels for the first three resonances are indicated above Figure 3a.
dashed vertical lines in Figure 2. We discuss the effect of the boundaries and azimuthal symmetry in section 4.

[14] The equatorial mass density $\rho$ is shown in Figure 2a (thick solid curve) and also in Figure 4a. Along the field lines, $\rho$ is assumed to vary with radius $R$ as $R^{0.5}$, a dependence consistent with the results of Goldstein et al. [2001]. There is a small plasma pressure in the simulation to prevent nonlinear instability of the wave, but this pressure has a negligible effect on the results (plasma beta $\ll 1$). The magnetic field is dipolar, but with the dipole moment adjusted so that the magnetic field at the position of CRRES is equal to the observed value, 107 nT (this makes the field correct in the outer $L$ region which has the greatest effect on the cavity resonance frequencies).

[15] At the outer $L$ boundary an azimuthal perturbation (westward) of the electric field was applied in order to generate an inward compressional pulse. The time dependence of this pulse was $\sin(\pi t/\tau_0)^2$ for $0 \leq t \leq \tau_0 \sim 26s$. The boundary azimuthal electric field had the spatial dependence $\cos(\pi \tau_{Ai}/(2\tau_{Ai}^2))$, where $\tau_{Ai} = 0$ at the equator and $\tau_{Ai}$ is the value of $\tau_{Ai}$ at the ionospheric boundaries. (The argument of the cos function is $\mp \pi/2$ at the ionospheric boundaries so that the value of the function there is zero.) The perturbation electric field is small, so that the simulation results are effectively linear. After the time period of the initial perturbation the boundary condition at the outer $L$ boundary is that the electric field and velocity are 0. This is a no-slip perfect conductor boundary condition. At that point we continue to run the simulation for 2600 s (43 min). Since the boundaries are perfectly conducting, there is no loss of energy.

[16] After storing time series of the azimuthal electric field $E_\phi$ at the position of CRRES and the kinetic energy, we take a Fourier transform in time to obtain the power spectra. These are plotted in Figures 3c and 3d, respectively. (Because the kinetic energy is quadratic in the field amplitude and therefore has doubled frequency, we have divided the frequencies by 2 for the kinetic energy power spectra in order to get the frequency of the modes.) The vertical dashed lines in Figure 3 are drawn at the frequencies corresponding to the peaks in kinetic energy. The mode labels (1, 2, or 3) are given above Figure 3a. Note that the theoretical peaks in Figure 3c and 3d agree very well with the observed power spectra and coherence peaks (Figures 3a and 3b, respectively). (Based on the coherence, the observed peaks appear to be slightly higher than the theoretical peaks, especially for mode 3 marked “3” in Figure 3b, though the power spectra appear to give frequencies closer to the theoretical ones.) Comparing Figures 3a and 3c, the peaks in the power spectra of the observed $E_y$ are broader than the peaks for the simulation $E_{\phi}$, but this is easily explained by the finite lifetime of the observed

Figure 4. (opposite) (a) Simulation mass density $\rho$ and contours of simulation (b, d, f) $E_\phi$ and (c, e, g) kinetic energy density $K$ versus $X$ and $Z$. Figures 4b and 4c are for mode 1, Figures 4d and 4e are for mode 2, and Figures 4f and 4g are for mode 3. Negative contour levels are dashed; the sign of $K$ is the sign of the radial velocity. The asterisk indicates the position of CRRES at the time of the Pi2.
waves. Note also that the dominant power is in the fundamental mode in both the observations and simulation; apart from this, we could have used an eigenvalue code to get the results presented in this paper. (The relative power in the fundamental increases with the timescale of the input perturbation $t_0$ [Lee and Lysak, 1999].) Because of the approximations involved in our theoretical model (discussed in section 4), we do not wish to emphasize the near exact agreement of theoretical and observed frequencies too much. However, it is clear that at least rough agreement of the frequencies has been demonstrated, and this fact does support the cavity resonance explanation of compressional Pi2s.

[17] After finding the resonance frequencies, we reran the simulation doing a time Fourier transform for each resonance frequency at each point in space. By this procedure, we calculated the first three eigenfunctions, and these are plotted in Figure 4. (These plots are similar to those of Ionaga et al. [1992].)

[18] The coordinates for the contour plots are SM, except that $X$ is measured radially outward (it is the negative of the SM $X$ in the midnight meridian). Figure 4b shows contours of the azimuthal electric field $E_y$ for the lowest frequency mode 1 (the fundamental). While $E_y$ is distributed over the simulation domain, the kinetic energy is more localized. In Figure 4c, contours of the kinetic energy density $K$ are plotted, and here we see that the kinetic energy is mostly localized to the enhanced density region between $L = 4.3$ and 6.8. Similar contour plots are presented for mode 2 in Figures 4d and 4e and for mode 3 in Figures 4f and 4g. The kinetic energy density $K$ has the sign of the $L$ component of velocity, and negative contour levels are indicated by dashed curves. Clearly, mode 2 has second harmonic structure in the radial direction, while mode 3 has third harmonic spatial structure in the parallel direction. (The simulation only produces eigenmodes that are symmetric about the equator since the input compressional pulse has this symmetry.)

4. Discussion

[19] While our simulation has demonstrated excellent agreement between the observed and theoretical cavity resonance frequencies, there are a number of approximations which need to be addressed. First, we have computed time stationary eigenmodes whereas observed Pi2 pulsations often have a complex waveform. To provide an example, the observed waveform shown in Figure 1 appears to consist of a series of heavily damped one- or two-cycle oscillations, as noted by Takahashi et al. [2001]. A possible interpretation is that the plasmasphere is a low-Q (heavily damped) cavity and that each oscillation was simulated by a harmonic spatial structure in the parallel direction. As a rough approximation, we would expect the azimuthal mode number to affect the resonant frequencies like $f = \sqrt{f_0^2 + \left|f_{max}(2n\pi)\right|^2}$. Time-dependent simulations typically show that most energy from a boundary pulse goes to low azimuthal number modes, $m = 1–3$ [Pekrides et al., 1997; Denton et al., 2000]. To evaluate this form we use $R = 5 \sqrt{R_x}$ at which location the mode kinetic energy is maximized (Figure 4). The largest effect would be on the fundamental frequency using $m = 3$; however, for this $m$ the above formula for $f$ yields only a 10% increase in frequency, which would not significantly change the agreement between observed and theoretical frequencies (it would slightly improve the agreement with the coherence peaks).

[20] This interpretation is reasonable in light of a simulation conducted by Fujita et al. [2001] using an absorbing outer boundary and a dissipative ionosphere. The simulation showed that the plasmasphere sustains well-defined trapped fast mode oscillations even when the ionospheric dissipation is strong (see their Figure 2). In a simulation run for which the height-integrated ionospheric Pedersen conductivity was assumed to be low (0.16 mho, corresponding to a nighttime condition), a virtual resonance was evident in the plasmasphere even though it diminished in approximately two cycles. Despite strong damping, the frequency of the resonance changed little from another run which used a much higher Pedersen conductivity (16 mho) and produced a long-lasting resonance as expected. Clearly, the boundary conditions for the plasmaspheric cavity as well as the source characteristics need to be very realistic for simulations to produce waveforms that match the observations, but the frequencies appear to be less sensitive to such simulation characteristics.

[21] Next, we have assumed azimuthal symmetry using the plasma density profile measured by CRRES on the outbound leg of the orbit (during which the Pi2 event was observed). We know that the plasmasphere can exhibit a great deal of azimuthal structure [Sandel et al., 2001]. We have some evidence of such structure in Figure 2a; the extended region of the plasmasphere outside $L = 4.2$ (which has greater extent on the outbound leg of the orbit (MLT = 20–23.4) than on the inbound leg (MLT = 23.4–4.5). (Other than this, the density features are remarkably symmetric with respect to the inward and outward legs of the orbit [see Carpenter et al., 1993].) On the other hand, the CRRES field measurements indicate that the wave energy is only localized in the radial direction. Since the wave energy has not had time to equilibrate in the azimuthal direction, it is reasonable to use an equilibrium based on the local time where the waves are observed. While three dimensionality in the plasma equilibrium does not seem to be required to simulate the cavity mode frequencies, our results do depend strongly on the use of the density profile on the outbound leg of the orbit, and in this sense, the azimuthal structure (particular density profile on the outbound leg) is important.

[22] Next, the wave perturbation cannot be azimuthally symmetric about the entire Earth. As a rough approximation, we would expect the azimuthal mode number to affect the resonant frequencies like $f = \sqrt{f_0^2 + \left|f_{max}(2n\pi)\right|^2}$. Time-dependent simulations typically show that most energy from a boundary pulse goes to low azimuthal number modes, $m = 1–3$ [Pekrides et al., 1997; Denton et al., 2000]. To evaluate this form we use $R = 5 \sqrt{R_x}$ at which location the mode kinetic energy is maximized (Figure 4). The largest effect would be on the fundamental frequency using $m = 3$; however, for this $m$ the above formula for $f$ yields only a 10% increase in frequency, which would not significantly change the agreement between observed and theoretical frequencies (it would slightly improve the agreement with the coherence peaks).

[23] Next, we consider the effect of the boundary conditions. The inner boundary of the simulation would better be placed at $L = 2$, at which $V_A$ is quite large (Figure 2b). With a perfectly conducting boundary, the frequency of the cavity mode is determined by the time for energy to propagate back and forth across the oscillatory domain.
Table 1. Frequency Ratios

<table>
<thead>
<tr>
<th>Study</th>
<th>$f_3/f_1$</th>
<th>$f_5/f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This paper (CRRES data)</td>
<td>1.8 ± 0.5</td>
<td>2.5 ± 0.7</td>
</tr>
<tr>
<td>This paper (theoretical)</td>
<td>1.8</td>
<td>2.4</td>
</tr>
<tr>
<td>Lin et al. [1991] (observations)</td>
<td>1.5 ± 0.3</td>
<td>2.0 ± 0.4</td>
</tr>
<tr>
<td>Nose´ [1999] (observations)</td>
<td>1.7 ± 0.5</td>
<td>2.3 ± 0.7</td>
</tr>
<tr>
<td>Cheng et al. [2000] (observations)</td>
<td>1.7 ± 0.1</td>
<td>2.3 ± 0.1</td>
</tr>
<tr>
<td>Lin et al. [1991] (theory)</td>
<td>1.8</td>
<td></td>
</tr>
<tr>
<td>Itonaga et al. [1992] (theory)</td>
<td>1.6</td>
<td>2.2</td>
</tr>
<tr>
<td>Itonaga et al. [1997] (theory)</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>Fujita et al. [2000] (theory)</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>Cheng et al. [2000] (theory)</td>
<td>1.7</td>
<td>2.4</td>
</tr>
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</table>

(within the WKB approximation). Therefore, as a rough approximation, we expect the frequencies to go like the inverse of the L shell Alfvén crossing time $\tau_A$, but Figure 2c shows that placing the boundary at $L = 2$ would make only a 3% difference in $\tau_A$. As discussed in the last paragraph, the fact that the perturbation is axisymmetric ($m = 0$) does not directly alter the frequencies significantly, but there can be an indirect effect. For nonaxisymmetric perturbations the plasmapause can serve as a potential wall which confines energy within the plasmasphere regardless of the simulation outer boundary [Lee, 1996; Lee and Lysak, 1990]. On the other hand, for $m = 0$ the eigenmodes will extend out to the simulation outer boundary. Because of this, we place our simulation outer boundary close to the plasmapause so that the frequencies will be characteristic of plasmaspheric modes rather than modes which have significant amplitude outside the plasmasphere (global modes). Of course, for the nightside magnetosphere a more accurate boundary condition is an open outer simulation boundary rather than a perfect conductor [Fujita and Glassmeier, 1995; Lee and Kim, 1999]. (Lee and Lysak [1999] use a combination of open and partially reflecting boundaries in their three-dimensional simulation.) With an open outer boundary the frequencies of plasmaspheric modes would be even less sensitive to the region outside the plasmasphere, since any energy leaking out beyond the plasmapause would be lost. [24] Because the plasmapause is not a perfect reflector, and the modes will be evanescent in the plasma trough, there will not necessarily be a node in the electric field exactly at the plasmapause [Lee and Kim, 1999]. The modes will be somewhat broader than the plasmasphere, and to represent this fact the best location for the boundary is slightly outside the plasmapause (such as the location we used at $L = 7.3$). Simulations with $m \neq 0$ and an open outer boundary would be useful for getting a better measure of the mode frequencies, but our outer boundary condition should give roughly the correct results for the plasmaspheric modes. Finally, studies have shown that ionospheric resistivity does not greatly affect the real frequency of the modes [e.g., Fujita et al., 2001], as mentioned in the second paragraph of this section.

[25] Next, we assumed that the plasma beta is approximately zero. The cavity resonance frequencies are determined by the speed within the cavity, and this will be affected by plasma pressure. Unfortunately, CRRES plasma data are not available, but the plasmasphere typically has low plasma beta, so our approximation is reasonable. (Note also that $Dst$ was quite low during 1991 day 48, between 0 and $-10$, so there is no evidence for injection of many high-energy particles into the inner magnetosphere.) We also assumed that the mass density was equal to the electron density observed by CRRES (Figure 2a). Again, heavy ion data are not available for this time segment, but based on the mass density model of Gallagher et al. [2000], the mass density should only be $\sim 15%$ larger than the electron density, and this would lead to only a $7\%$ difference in frequency. In summary, our simulation technique involves many approximations, so that we should not expect to exactly calculate the plasmaspheric cavity frequencies. Nevertheless, because the approximations are in every case reasonable, the fact that we do find agreement between observed and theoretical frequencies does give support to the cavity resonance explanation of plasmaspheric compressional $P \bar{i}2$ pulsations.

[26] From Figure 3d our theoretical frequency for the fundamental plasmaspheric cavity resonance is 7.6 mHz. Based on the Alfvén crossing time $\tau_A$ (not normalized), the fundamental frequency for a mode localized between $L = 2.5$ and $L = 7.3$ (simulation boundaries) would be $1/(2 (\tau_A(L = 7.3) - \tau_A(L = 2.5))) = 6.3$ mHz, $\sim 80\%$ of the simulation value. Values of the frequency ratios $f_3/f_1$ and $f_5/f_3$ from our simulation are given in Table 1. The value of $f_3/f_1$ expected based on the crossing time would be 2, as compared with the simulation value 1.8. The ratio $f_5/f_3$ cannot be easily predicted from the equatorial crossing time, since mode 3 has structure along the field line direction rather than across $L$ shells (Figures 4f and 4g).

[27] There have been previous studies of frequency ratios based on observations [Lin et al., 1991; Nose´, 1999; Cheng et al. 2000] and theory using plasmaspheric density models [Lin et al., 1991; Itonaga et al. 1992, 1997; Fujita et al., 2000; Cheng et al. 2000]. Values from the studies are shown in Table 1. (The simulation of Allan et al. [1986] emphasizes modes which have large amplitude in the plasma trough, that is, global modes.)

[28] Examination of Table 1 shows that our results are generally consistent with these studies despite the fact that our density profile includes two regions with enhanced density (Figure 2a).

5. Conclusions

[29] The theoretical frequencies from our MHD simulation of the 17 February 1991 $P \bar{i}2$ event observed by CRRES [Takahashi et al., 2001] are in good agreement with the observed frequencies, and this supports the cavity resonance mechanism as an explanation for the observations. (The near exact agreement should not be overemphasized since there are many possible errors, but certainly our results show that the frequencies agree at least roughly.) The observed and theoretical harmonic frequency ratios $f_3/f_1$ and $f_5/f_3$ were in rough agreement with earlier studies. Comparison of the outbound and inbound legs of the CRRES orbit on 17 February 1991 shows that the density on the outbound leg at MLT = 20–23.4 (on which the compressional $P \bar{i}2$s occurred) was different than that on the inbound leg at MLT = 23.4–4.5. On the outbound leg an enhanced density region beyond the region of the nominal plasmapause (which may have been part of a plasmaspheric tail [Sandel et al., 2001]) played an important role in determining the cavity resonance frequencies. Observations
by the IMAGE spacecraft have revealed that there is often a great deal of azimuthal structure in the plasmasphere (including such tails). In our axisymmetric simulation the effect of this azimuthal structure is represented in our choice of the outbound density profile versus the inbound profile. Such structure may play an important role in determining observed Pi2 frequencies.

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