The Proton Concentration in the Vicinity of the Io Plasma Torus

R. L. Tokar and D. A. Gurnett

The University of Iowa, Department of Physics and Astronomy, Iowa City, Iowa 52242

F. Bagental

The Massachusetts Institute of Technology, Center for Space Research, Cambridge, Massachusetts 02139

This paper uses an improved model for the plasma distribution in Jupiter's magnetosphere to determine the light ion concentration in the vicinity of the Io plasma torus from whistler dispersion measurements. The model used assumes that the plasma is in diffusive equilibrium under the action of centrifugal, gravitational, and ambipolar electric field forces. The study provides an estimate of the plasma concentration at intermediate and high latitudes along field lines through the Io plasma torus. The method employed is to combine the Voyager 1 plasma wave instrument whistler observations with the Voyager 1 plasma instrument heavy ion charge concentrations throughout the Io torus to determine the light ion charge concentration along the whistler propagation paths. Because the light ion source is probably Jupiter's ionosphere and because Jupiter's atmosphere is primarily hydrogen, the light ions are taken to be protons. Whistlers at 13 L values between L = 3.25 and L = 5.85 are analyzed. Values of NL^2, the total number of ions per unit L multiplied by L^2, are calculated and the ratio NL^2 (protons)/NL^2 (electrons) is found to have an average value of 0.2. This ratio is used to give a rough estimate for the ionospheric source strength.

1. Introduction

The Voyager 1 plasma wave instrument observation of lightning-generated whistlers taken during the March, 1979 encounter of Jupiter have been employed in numerous studies involving Jupiter's inner magnetosphere [Gurnett et al., 1979; Menietti and Gurnett, 1980; Gurnett et al., 1981; Tokar et al., 1982]. Because the observed dispersion of the whistler signal is related to the electron concentration encountered by the wave as it travels from the ionosphere to the spacecraft, the whistler observations can be used to investigate the plasma concentration in regions not directly sampled by the spacecraft. In Tokar et al. [1982] the Voyager whistler observations were combined with heavy ion charged particle measurements in the Io torus [Bagental and Sullivan, 1981], to determine the light ion charge concentration along the whistler propagation paths. In the Tokar et al. [1982] study, simple models were used for the plasma distribution along the propagation paths. In this paper, the procedure is repeated by using an improved model for the plasma distribution in the inner magnetosphere. The model adopted, which treats a plasma in diffusive equilibrium under the action of gravitational, centrifugal, and ambipolar electric field forces, is similar to the models used by Angerami and Thomas [1964] for the earth's ionosphere and magnetosphere and by Bagental and Sullivan [1981] for Jupiter's inner magnetosphere. Although the results obtained in this paper using the diffusive equilibrium model are qualitatively similar to the results given in Tokar et al. [1982], the procedure followed possesses a self-consistency absent in the Tokar et al. [1982] study. The cost of obtaining these results is the added computational difficulty associated with a self-consistent treatment and the absence of a simple analytical representation for the plasma distribution.

The observations used in this work were taken by the Voyager 1 plasma and plasma wave instruments, descriptions of which can be found in Bridge et al. [1977] and Scarf and Gurnett [1977], respectively.

2. Whistler Theory

Observations of whistlers at earth and the theory of whistlers is described in detail by Helliwell [1965]. Because the whistler theory relevant to the problem at hand has recently been discussed by Tokar et al. [1982], only a brief summary will be given here.

Whistlers detected by the Voyager 1 plasma wave experiment exhibit the familiar decrease in frequency with increasing time which produces the whistling tone characteristic of lightning-generated whistlers. The frequency-time structure follows the relation given by Eckersley [1935]:

\[ t = \frac{1}{c} \int n_e ds \]

with

\[ n_e = \frac{1 + \frac{1}{2} \frac{f_p}{f_e} (f_e - f) \frac{f_E}{f_e}}{1 + \left( \frac{f_p}{f_e} (f_e - f) \frac{f_E}{f_e} \right)^{1/2}} \]  

(1)

In (1), t is the arrival time for frequency f, f_p, and f_e are the electron plasma frequency and the electron gyrofrequency, and the integral is along the magnetic field line from the ionosphere to the spacecraft. The field-aligned propagation assumption does not introduce a significant error because the group travel time of the whistler is relatively insensitive to deviations of the wave vector away from the magnetic field line [Helliwell, 1965]. The assumption of field-aligned propagation is also supported by the ray tracing calculations.
of Menietti and Gurnett [1980]. Because the whistler observations were made away from the equatorial region, the hemisphere from which the whistler originated must be determined. This problem is resolved by the fact that for one hemisphere the calculated dispersion, using the plasma instrument heavy ion concentration as the electron concentration, is greater than the observed dispersion. To calculate \( f_s \), the magnetic field is assumed to be dipolar with the dipole moment adjusted to correspond closely with the Voyager magnetometer measurements [Ness et al., 1979].

The reception time integral of (1) can be used to calculate the whistler frequency-time structure if the plasma frequency (the electron concentration) along the propagation path is known. The free parameter in a model for the electron concentration along the field line can be defined through the requirement that the observed and calculated dispersions agree. With this in mind, the Voyager plasma instrument heavy ion (8 \( \leq A/Z \leq 64 \), where \( A/Z \) is the ion mass to charge ratio) concentrations have been supplemented with a light ion (\( A/Z < 8 \)) concentration and extrapolated away from the spacecraft trajectory by using a diffusive equilibrium model. Assuming local charge neutrality, the electron concentration is then known along the whistler propagation path. The number of light ions along the propagation path is the free parameter to be adjusted so that the computed and observed whistler dispersions agree.

Because this study is concerned with determining the light ion concentration profile, which, when added to the Io torus heavy ion concentration profile, produces the observed whistler dispersion, it is desirable to obtain a measure of the light ion contribution to the whistler dispersion. This measure is provided in Figure 1, which shows \( n_e \) as a function of path length along the field line \( s \) for the two extreme frequencies of 1 and 10 kHz. Figure 1 corresponds to an electron concentration profile along the field line typical of plasma distributions along field lines through the Io plasma torus. Referring to (1) we observe that if the whistler is received by a spacecraft located at the magnetic equator, the difference in the arrival time for the two frequencies is proportional to the shaded area in Figure 1. From \( t = D/\sqrt{f} + t_0 \), it is seen that this area is essentially a measure of the whistler dispersion. In the case shown, the light ion contribution to the total ion concentration ranges from about 10% at the magnetic equator to 100% for \( s \) greater than 2 \( R_J \). It is evident that the primary contribution to the whistler dispersion occurs at low latitudes where the heavy ions are the dominant component of the ion population. Nevertheless, Figure 1 indicates that a significant portion of the observed whistler dispersion is due to propagation in the intermediate and high-latitude regions. Because the light ions are the sole component of the ion population, in these regions the light ion profile determines the electron plasma frequency profile for use in the evaluation of the group index of refraction.

3. DIFFUSIVE EQUILIBRIUM MODEL

To find the plasma distribution that gives the best fit to the whistler dispersion, a prescription must be adopted for extrapolating the ion concentration along the magnetic field line from the whistler observation point to the ionosphere. We use a model that describes a plasma in diffusive equilibrium. The model is similar to that used by Bagenal and Sullivan [1981] to extrapolate the heavy ion concentration measured by the plasma instrument through the Io torus. Angerami and Thomas [1964] first developed models of this type in their study of the plasma distribution in earth's ionosphere and magnetosphere.

The tilted-dipole geometry of the inner magnetosphere used in this study is shown in Figure 2. The angle \( \alpha \) between the rotational and magnetic equators is a function of longitude and attains its maximum magnitude of 9.6° at system III longitudes of 202° and 22°. The angle \( \beta \) is the angle between the rotational equator and the centrifugal symmetry surface defined by Hill et al. [1974]. In Jupiter's inner magnetosphere near the equatorial plane it is well known that the centrifugal force plays the major role in determining the equilibrium plasma configuration. Consequently, the position of the centrifugal symmetry surface is near the point at which the field-aligned component of the centrifugal force vanishes. It is easily shown that this requirement leads to the equation \( \sin \alpha = 3 \sin \beta (1 + 4 \tan^2 \beta \beta)^{1/2} \), which determines \( \beta \) given \( \alpha \). Since we are dealing with small angles, \( \beta \approx \alpha/3 \).

Figure 2 shows a plasma parcel constrained to the magnetic field line and at the position \( (r, \theta) \). To derive the diffusive equilibrium model, we assume collisions have brought this parcel and the surrounding plasma to thermal equilibrium. Under such circumstances we can apply the Gibbs distribution from classical statistical mechanics to determine the average value of the concentration in the plasma parcel. (Alternative derivations of this model are given in Angerami and Thomas [1964] and Bagenal and Sullivan [1981].) Applying the Gibbs distribution, the following is ascertained: if \( n(s) \) denotes the concentration at a distance \( s \) above the magnetic equator along the field line, then \( n(s) = n_0 \exp -(E(s)/kT) \), where \( E(s) \) is the potential energy at position \( s \), \( T \) is the plasma temperature, and \( k \) is Boltzman's constant. In the

![Fig. 1](image_url)
model used in this study, the potential energy $E(s)$ is calculated by using the field-aligned components of the centrifugal, gravitational, and ambipolar electric field forces. The contribution of the magnetic mirror force to the energy is neglected as the particle pitch angle distributions are not known. As discussed by Bagental and Sullivan [1981], the error introduced is small for the heavy ions in the torus region. The error is greatest for the hot component of the electron population, a component present only for $L > 5.5$. Scudder et al. [1981] quote the values $T_{\text{Hot}} = 625$ eV and $n_{\text{Hot}}/n_{\text{Cold}} = 0.0002$ from the Voyager electron measurements at 5.5 $R_J$. Because only a small range in $L$ is considered in this paper, these values are adopted for all whistlers with $L > 5.5$. It should be noted that the hot electrons do not play a large part in the plasma distribution due to their low concentrations.

The rotational and gravitational potential energies that contribute to $E(s)$ are determined by the field-aligned components of the centrifugal force $F_{\text{c}}$ and the gravitational force $F_g$; see Figure 2. At low latitudes the centrifugal force is dominant, while at high latitudes the gravitational force is dominant. For example, on an $L = 6$ field line for $\alpha = 9.6$ degrees the field-aligned components of the centrifugal and the gravitational force are equal at $\theta = 49^\circ$. This angle corresponds to a field-aligned distance of 1.7 $R_J$ above Jupiter’s surface. The gravitational force has a negligible effect on the plasma distribution in the torus region ($\theta \leq 20^\circ$) but can maintain a nonzero concentration in the intermediate and high latitude portions of the whistler propagation path; see Figure 3.

To understand the origin of the ambipolar electric field, consider the original establishment of a steady state inner magnetosphere. As heavy ion-electron pairs are created (e.g., by electron impact ionization of neutrals near Io) the centrifugal forces will confine the heavy ions to the equatorial region, while the electrons, due to their small mass, are less affected and can distribute uniformly along the field line. The resulting charge separation creates an electric field that pulls the ions away from the equatorial region and the electrons toward the equatorial region. The electric field that remains when the steady state situation is reached is called an ambipolar electric field. A schematic drawing of the charge layers which produce the field is shown in Figure 2.

Fig. 3. This figure shows the ion concentration profiles obtained for the whistler observed at 0957 by using the diffusive equilibrium model. The resulting electron concentration reproduces the observed whistler dispersion. The heavy ions are closely confined to the centrifugal symmetry surface region, while the protons are characterized by a large scale height. The profiles do not possess mirror symmetry about the centrifugal symmetry surface due to the angle between the rotational and magnetic axes. The position where the field-aligned components of the gravitational and centrifugal forces are equal is shown, as is the spacecraft position at the whistler observation time.
The deviation from local charge neutrality is slight but non-zero along the field line. When employing the Gibbs distribution to derive the diffusive equilibrium model, the ambipolar electric field’s contribution to the potential energy at the plasma parcel is included.

From the Gibbs distribution, the diffusive equilibrium model for ion species $i$ is

$$n_i(s) = n_{i0} \exp \left( -\frac{1}{kT_i} \left( \Delta \phi_{i,1} + \Delta \phi_{i,2} + Z_i e \Delta \phi \right) \right)$$

(2)

where $n_{i0}$ is the reference concentration at $s_0$, $\Delta \phi_{i,1}$ and $\Delta \phi_{i,2}$ are the changes in the centrifugal and gravitational potential energies for species $i$ referenced to $s_0$, respectively, and $\Delta \phi$ is the change in the ambipolar electrostatic potential energy referenced to $s_0$. The ion temperature is $T_i$ and the ion charge state is $Z_i$ (e.g., for O$^2^+$, $Z = 2$). The heavy ion temperatures, which range from 10 to 30 eV, and the reference concentrations are those measured by the plasma instrument [Bagental and Sullivan, 1981]. For simplicity, the species temperature is assumed to remain constant along the field line. Similarly, both the hot and cold electron components are described by an equation like

$$n_e(s) = n_{e0} \exp \left( -\frac{1}{kT_e} \left( \Delta \phi_{e,1} + \Delta \phi_{e,2} - e \Delta \phi \right) \right)$$

(3)

These equations are coupled by the local charge neutrality assumption

$$n_e(s) = \sum_i Z_i n_i(s)$$

(4)

Equations (2), (3), and (4) are combined with the whistler theory to reproduce the frequency-time structure of the observed whistler. The procedure is to vary $n_{i0}$ for the light ions until the computed dispersion agrees with the observed dispersion. Because Io is only known to contribute heavy ions, the light ions probably originate in Jupiter’s ionosphere. The light ions are assumed to be protons with a temperature of 20 eV. While the whistler theory can be used to define simultaneously more than one light ion species if the relative concentrations are known, one light ion species, namely H$^+$, is consistent with an ionospheric source in Jupiter’s atmosphere that is known to be primarily hydrogen. The 20 eV temperature is reasonable because, again, considering only the gravitational and centrifugal forces, on an $L = 6$ field line for an $\alpha = 9.6$ an ionospheric proton with the requisite 9 eV of field-aligned energy will escape the ionosphere and acquire a field-aligned energy of 20 eV when it reaches the equatorial region. Pitch angle scattering can serve to prevent the proton from being absorbed in the opposing ionosphere. Proton energies higher than 20 eV would not significantly affect the results obtained in this study as the light ion profile for a 20 eV temperature is characterized by a large scale height; see Figure 3.

Before presenting the results the following unavoidable defect in the diffusive equilibrium model is mentioned: only ion concentrations corresponding to observed ions can be extrapolated away from the spacecraft trajectory. Heavy ions can remain undetected if the spacecraft is sufficiently far away from the centrifugal symmetry surface. As an example, at a few positions where whistlers were observed the scale height of SO$_2^+$, calculated using $H = (2kT/3m\omega^2)^{1/2}$ where $T$, $m$, and $\omega$ are the species temperature, species mass, and angular rotation rate of Jupiter, respectively, (e.g., see Bagental et al. [1980]), is smaller than the distance of the spacecraft from the centrifugal symmetry surface. In this regard, the joining of the whistler observations and the heavy ion observations has served to demonstrate that the plasma instrument has detected most species of heavy ions. If these were not true, the number of protons required to reproduce the observed dispersions for propagation paths containing and not containing the bulk of the Io torus would show striking differences.

4. Results

The whistler observations have been combined with the heavy ion measurements by using the diffusive equilibrium model to determine the proton concentrations for 13 groups of whistlers, corresponding to 61 individual whistlers, with $L$ values between 5.25 and 5.85 $R_J$. The whistlers are grouped because whistlers occurring in rapid succession often have nearly equal dispersions. For each whistler the dispersion is determined by using $\tau = D/\sqrt{\Delta f}$ + $\tau_o$. The individual whistler dispersions are then used to calculate an average dispersion $D$ and a dispersion standard deviation $\Delta D$ for each group. Table 1 summarizes the whistler parameters. The first 11 whistler groups are characterized by dispersions from about 40 to 85 s Hz$^{-1/2}$, while the last two groups have dispersions on the order of 240 to 300 s Hz$^{-1/2}$. The small dispersion groups correspond to northern hemisphere sources, while the large dispersion groups correspond to southern hemisphere sources. A north (south) source has a propagation path that does not (does) cross the centrifugal symmetry surface. This collection of whistlers differs from the multigroups treated in Tokar et al. [1982] as follows: the first three groups, corresponding to 27 whistlers, were omitted from the previous study because their propagation paths did not lie in the heavy ion concentration grid, are included in this study; the group observed at 0937 is not included here because of an inconsistency in the plasma instrument measurements at that time; and the final three groups, observed at 1505, 1507, and 1511, with $L$ values of 6.04, 6.06, and 6.10, respectively, are omitted because the plasma instrument could only obtain good data for these $L$ values on the inbound leg of the trajectory when the spacecraft was at different longitudes. These whistlers are omitted because, as will be mentioned later, a self-consistent employment of the diffusive equilibrium model does not allow a plasma number density measurement for a given $L$ value and a given longitude to be used at a different longitude. However, an estimate of the ion concentration at these $L$ values can be obtained from the results of the Tokar et al. [1982] study. This estimate will be given shortly. Because the final three groups were omitted, all whistler observations and plasma instrument measurements used in this paper were made at the same time.

The results obtained for the group observed at 0957 are shown in Figure 3. The lightning stroke that generated this group of whistlers was located in the northern hemisphere. The spacecraft at this time was located slightly north of the centrifugal symmetry surface. As can be seen, the five heavy ion species detected by the plasma instrument are confined by the centrifugal force to a region near the centrifugal symmetry surface. The close confinement is due to the fact
that this whistler group lies in the cold region of the torus ($L \lesssim 5.6$). In the warm torus ($L \gtrsim 5.6$), the heavy ion species can extend as far as 2 $R_J$ away from the centrifugal symmetry surface. The proton profile which, when added to the heavy ion profile, reproduces the whistler frequency-time structure is also shown in Figure 3. The proton profile is characterized by a large scale height and is influenced by the gravitational force at intermediate and high latitudes. Note that the profiles do not possess mirror symmetry about the centrifugal symmetry surface. This is due to the tilt of the rotational and magnetic axes.

All the results obtained are summarized in the last two figures. In Figure 4, the concentrations are presented in the form of a three-dimensional surface plot. The electron concentrations are shown as a function of $s$, the distance along the field line from the magnetic equator, for the 13 $L$ values in Table 1. For all 13 groups, points of equal distance away from the magnetic equator are connected by straight lines. The high heavy ion concentration near the centrifugal symmetry surface and the proton concentration at intermediate and high $s$ values are clearly visible. The figure provides a visual indication of the variation with $L$ in the proton concentration in the intermediate and high-latitude regions.

It should be noted that the group at $L = 5.68$, which required no light ions to reproduce the observed dispersion (see the next paragraph), is given a light ion concentration of 1 cm$^{-3}$ outside of the Io torus to avoid computational difficulty.

In Figure 5 the values of $NL^2$, the total number of ions per unit $L$ multiplied by $L^2$, are shown for the electrons and the protons at the 13 $L$ values analyzed. The variation of $NL^2$ with $L$ is an important quantity in radial diffusion studies (e.g., see Richardson and Siscoe [1981]). $NL^2$ is calculated from the column density $N_e$ of ions or electrons as follows:

$$N_e = B_s f(nB)ds \quad NL^2 = 2\pi(R^2L^3N_e)$$  \hspace{1cm} (5)

In (5) the integral is along the magnetic field line at the flux

![Fig. 5. In this figure the values of $NL^2$, calculated by using (5), are shown for the electrons and protons as a function of radial distance. The error bars correspond to the concentrations obtained over the range $D - \Delta D$ to $D + \Delta D$ in the group dispersions. For average group dispersion, the ratio $NL^2$(protons)//$NL^2$(electrons) ranges from about 0 to 0.4, with an average value of 0.2. It should be noted that $NL^2$ is calculated by assuming azimuthal symmetry. With these values of $NL^2$ and (5) the column density $N_e$ at the longitude where the whistler was observed can be determined. The dotted line between 5.64 and 5.72 $R_J$ corresponds to the whistler group at 0957, which required no light ions along the whistler propagation path to reproduce the observed dispersion. The value of $NL^2$ at $L = 6.05$ were not calculated by using the diffusive equilibrium model. These values are estimates obtained from Tokar et al. [1982].](image-url)
tube, $B_0$ is the equatorial magnetic field strength at the flux tube, and $n$ and $B$ are the particle concentrations and the magnetic field strength, respectively, along the field line. Equation (5) is derived from the flux tube definition, $B_0 A = \text{constant}$. Note that the symbols $R_I$, $L$, and $N_i$ have units cm$/R_I$, $R_I$, and cm$^{-2}$, respectively, so $NL^2$ has dimensions of Jovian radii. In Figure 5, the error bars correspond to the concentrations obtained over the range $\hat{D} - \Delta D$ to $\hat{D} + \Delta D$ in the group dispersions. $NL^2$ values for the average group dispersions are connected by straight lines. The dotted line in the proton values between $L = 5.64$ and $L = 5.72$ corresponds to the whistler group at $L = 5.68$, which required no protons to reproduce the observed dispersion. At this time, an explanation for the unusual result at $L = 5.68$ is not known. For completeness, the estimate $5 \times 10^{53} R_I$ for the proton $NL^2$ value at $L = 6.05$, corresponding to the three whistler groups, is included in Figure 5. This estimate is consistent with the $NL^2$ values calculated in Tokar et al. [1982] and indicates an increase in the proton $NL^2$ value from $L = 5.85$ to $L = 6.05$. The electron $NL^2$ value at $L = 6.05$ is also taken from Tokar et al. [1982].

The values of $NL^2$ shown in Figure 5 are calculated by assuming azimuthal symmetry. It should be noted that the diffusive equilibrium model and plasma number density measurements for a given $L$ value and a given longitude cannot be used to calculate the plasma concentration at a different longitude because the angle $\alpha$ is a function of longitude; see Figure 2. Plasma number density measurements at one point on a magnetic field line for a given $L$ value and a given longitude can be extrapolated to the centrifugal symmetry surface by using the diffusive equilibrium model. However, this concentration is only valid at the given longitude because the curvature of the dipole field line with respect to the centrifugal symmetry surface varies with longitude. Because this effect is small, the calculation of an azimuthally averaged value for $NL^2$ is justified. Using (5) and the values of $NL^2$ in Figure 5, the column density $N_i$ at the longitude of the whistler observation can be determined.

5. Discussion

As was previously mentioned, the light ions are probably protons that originate at Jupiter's ionosphere. If the dominant diffusive mechanism operating in the Io torus region is flux tube interchange and if the solar wind is assumed to contribute a negligible number of ions to the inner Jovian magnetosphere, the values of $NL^2$ shown in Figure 5 can be used to provide a rough estimate of the ionospheric source strength. From Figure 5 the ratio $NL^2$(protons)/$NL^2$(electrons) for average group dispersion varies from about 0 to 0.4 with an average value of 0.2. While a precise value is not known, a range in the value for the rate at which heavy ions are produced near Io is $10^{28-31}$ ions/s [Hill, 1979; Shemansky, 1980]. Using the average ratio for $NL^2$(protons)/$NL^2$(electrons), a rough estimate for the range in the ionospheric source strength is $2.5 \times 10^{27-31}$ ions/s. It is evident that this result is critically dependent on the value chosen for the heavy ion source strength. Because the precise value for the heavy ion source strength is a matter of discussion, the primary result here is that the ratio $NL^2$(protons)/$NL^2$(electrons) has the average value 0.2 over the range in $L$ of 5.25 to 5.85 $R_I$.

The results reported in this paper define the proton concentration that must be added to the Io plasma torus heavy ion concentration in order that the observed and calculated whistler dispersions agree. The plasma concentration profiles along the whistler propagation paths have been determined in a self consistent manner. Because the centrifugal forces are large, it is hard to imagine plasma concentration profiles that differ significantly from those obtained by using the diffusive equilibrium model. This study provides a good estimate of the intermediate and high latitude plasma concentration in the Io torus region. Further information on the plasma concentration in these regions will not be obtained until the Galileo spacecraft reaches Jupiter late in this decade.

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